# Definite Integration

## Question1

$\int_0^1 x \sin^{-1} x dx =$

### AP EAPCET 2025 - 26th May Morning Shift

**Options:**

A.

$\frac{\pi}{8}$

B.

$\frac{\pi}{4}$

C.

$\frac{\pi}{12}$

D.

$\frac{\pi}{3}$

**Answer: A**

## Solution:

$$I = \int_0^1 x \sin^{-1} x \cdot dx$$
$$\because \int uv \cdot dx = u \int v dx - \int \left(u' \int u dx\right) dx$$

Here, $u = \sin^{-1} x, v = x$

$$\because I=\left[\sin^{-1}x\int x\cdot dx-\int\left(\frac{1}{\sqrt{1-x^2}}\int x\cdot dx\right)dx\right]_0^1$$
$$=\left[\sin^{-1}x\cdot\frac{x^2}{2}-\int\frac{x^2}{2\sqrt{1-x^2}}dx\right]_0^1$$
$$=\left(\sin^{-1}(1)\cdot\frac{1}{2}-\sin^{-1}(0)\cdot 0\right)-\int_0^1\frac{x^2}{2\sqrt{1-x^2}}dx$$
$$=\frac{\pi}{4}-\frac{1}{2}\int_0^1\frac{x^2}{\sqrt{1-x^2}}dx$$

Let $x=\sin\theta\Rightarrow dx=\cos\theta\cdot d\theta$ at $x=1\Rightarrow\theta=\pi/2$ and $x=0\Rightarrow\theta=0$

$$=\frac{\pi}{4}-\frac{1}{2}\int_0^{\pi/2}\frac{\sin^2\theta\cdot\cos\theta}{\cos\theta}d\theta$$
$$=\frac{\pi}{4}-\frac{1}{2}\int_0^{\pi/2}\sin^2\theta\cdot d\theta$$
$$=\frac{\pi}{4}-\frac{1}{2}\int_0^{\pi/2}\left(\frac{1}{2}-\frac{\cos 2\theta}{2}\right)d\theta$$
$$=\frac{\pi}{4}-\frac{1}{2}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi/2}$$
$$=\frac{\pi}{4}-\frac{1}{2}\left[\frac{\pi}{4}-0-0\right]=\frac{\pi}{4}-\frac{\pi}{8}=\frac{\pi}{8}$$

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# Question2

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin(x-[x])dx=$

## Here $[x]$ is the greatest integer function

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**Options:**

A.

0

B.

$3(1-\cos 1)+\sin 2-\sin 1$

C.

$3(1-\cos 1)+\cos 2-\sin 1$

D.

$\cos 2-\sin 2$

**Answer: B**

## Solution:

$\because I=\int_{-\pi/2}^{\pi/2} \sin(x-[x]) dx$

for $x \in[-\pi/2,-1] \Rightarrow[x]=-2$

and $x-[x]=x+2$

for $x \in[-1,0) \Rightarrow[x]=-1$

and $x-[x]=x+1$

for $x \in[0,1] \Rightarrow[x]=0$ and $x-[x]=x$

for $x \in\left[1, \frac{\pi}{2}\right] \Rightarrow x-[x]=x-1$

$\therefore I=\int_{-\pi/2}^{-1} \sin(x+2 dx+\int_{-1}^{0} \sin(x+1) dx+\int_{0}^{1} \sin x \cdot dx+\int_{1}^{\pi/2} \sin(x-1) dx$

$=[-\cos(x+2)]_{-\pi/2}^{1}+[-\cos(x+1)]_{-1}^{0}+[-\cos x]_{0}^{1}+[-\cos(x-1)]_{1}^{\pi/2}$
$=-\cos(1)+\cos(2-\pi/2)-\cos(1)+\cos(0)-\cos(1)+\cos(0)-\cos(\pi/2-1)+\cos(0)$
$=-3\cos(1)+3+\cos(-(\pi/2-2))-\cos(\pi/2-1)$
$=3-3\cos(1)+\sin(2)-\sin(1)$

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# Question3

$\int_0^2 x^2(2-x)^5 dx=$

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**Options:**

A.

$\frac{128}{21}$

B.

$\frac{64}{7}$

C.

$\frac{32}{21}$

D.

$\frac{16}{7}$

**Answer: C**

## Solution:

$$\int_0^2 x^2(2-x)^5 dx$$
$$= \int_0^2 x^2\left(32-80x+80x^2-40x^3+10x^4-x^5\right)dx$$
$$= \left[\frac{32x^3}{3}-\frac{80x^4}{4}+\frac{80x^5}{5}-\frac{40x^6}{6}+\frac{10x^7}{7}-\frac{x^8}{8}\right]_0^2$$
$$= \left(\frac{32\times 8}{3}-20\times 16+16\times 32-\frac{20}{3}\times 64+\frac{10}{7}\times 128-\frac{256}{8}\right)-0$$
$$= \frac{256}{3}+16\times 12-\frac{1280}{3}+\frac{1280}{7}-32$$
$$= \frac{-1024}{3}+\frac{1280}{7}+160$$
$$= \frac{-7168+3840+3360}{21}=\frac{32}{21}$$

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# Question4

**If $f(x)=\max\left\{x^3-4, x^4-4\right\}$ and $g(x)=\min\left\{x^2, x^3\right\}$, then $\int_{-1}^{1}(f(x)-g(x))dx=$**

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**Options:**

A.

$-\frac{151}{20}$

B.

$\frac{9}{20}$

C.

$\frac{131}{22}$

D.

$-\frac{67}{9}$

**Answer: A**

## Solution:

$\because f(x)=\max\left\{x^{3}-4, x^{4}-4\right\},$

$g(x)=\min\left\{x^{2}, x^{3}\right\}$
at $x=-1 \Rightarrow x^{2}=1, x^{3}=-1$
$\Rightarrow \min\{1,-1\}=-1=(-1)^{3}$

Similarly, at $x=0.5, \Rightarrow x^{2}=0.25,$

$x^{3}=0.125$
$\Rightarrow \min\{0.25,0.125\}=0.125=(0.5)^{3}$

So, for $x \in[-1,1] \Rightarrow g(x)=x^{3}$

Similarly, analysis of $f(x)$

When, $x^{3}-4=x^{4}-4$

$\Rightarrow \quad x^{3}(x-1)=0$

So, $x=0,1$

for $x<0 \Rightarrow x^{4}-4>x^{3}-4$

for \$0x^4-x\$

thus, $f(x)=\begin{cases}x^{4}-4, & -1 \leq x \leq 0 \\ x^{3}-4, & 0<x \leq 1\end{cases}$

Therefore, $f(x)-g(x)$
$=\begin{cases}\left(x^{4}-4\right)-x^{3}=x^{4}-x^{3}-4, & -1 \leq x \leq 0 \\ \left(x^{3}-4\right)-x^{3}=-4, & 0<x \leq 1\end{cases}$
$\therefore \int_{-1}^{1}(f(x)-g(x)) d x=\int_{-1}^{0}\left(x^{4}-x^{3}-4\right) d x+\int_{0}^{1}-4 \cdot d x$

$$= \left[\frac{x^5}{5} - \frac{x^4}{4} - 4x\right]_{-1}^{0} - 4[x]_0^1$$

$$= \left[0 - \left(-\frac{1}{5} - \frac{1}{4} + 4\right) - 4(1) - 0\right]$$

$$= -8 + \frac{1}{5} + \frac{1}{4} = \frac{-160+5+4}{20} = \frac{-151}{20}$$

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# Question5

$$\int_0^\pi \left(\sin^5 x \cos^3 x + \sin^4 x \cos^4 x + \sin^3 x \cos^4 x\right) dx =$$

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**Options:**

A.

$\frac{873}{2240}$

B.

$\frac{3\pi}{128} + \frac{12}{35}$

C.

$\frac{1641}{4480}$

D.

$\frac{3\pi}{128} + \frac{4}{35}$

**Answer: D**

## Solution:

$$\text{Let } I=\int_0^\pi \left(\sin^5 x\cos^3 x+\sin^4 x\right.$$
$$\left.\cos^4 x+\sin^3 x\cos^4 x\right)dx \qquad \ldots(i)$$
$$\Rightarrow I=\int_0^\pi \left[\sin^5(\pi-x)\cos^3(\pi-x)\right.$$
$$+\sin^4(\pi-x)\cos^4(\pi-x)$$
$$\left.+\sin^3(\pi-x)\cdot\cos^4(\pi-x)\right]dx$$
$$\Rightarrow I=\int_0^\pi -\sin^5 x\cos^3 x+\sin^4 x\cos^4 x$$
$$+\sin^3 x\cos^4 x dx \qquad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I=2\int_0^\pi \left(\sin^4 x\cos^4 x+\sin^3 x\cos^4 x\right)dx$$
$$I=\int_0^\pi \frac{\sin^4 2x}{16}dx+\int_0^\pi \sin^3 x\cdot\cos^4 x dx$$

$$\cos x=t$$
$$\Rightarrow -\sin x dx=dt$$
$$=\frac{1}{16}\int_0^\pi \frac{(1-\cos 4x)^2}{4}dx+\int_1^{-1} t^4\cdot\left(1-t^2\right)(-dt)$$
$$=\frac{1}{64}\int_0^\pi \left(1-2\cos 4x+\cos^2 4x\right)dx.+\int_1^{-1} t^4\left(t^2-1\right)dt$$
$$=\frac{1}{64}\int_0^\pi \left(1-2\cos 4x+\frac{1+\cos 8x}{2}\right)dx+\left(\frac{t^7}{5}-\frac{t^5}{5}\right)_1^{-1}$$
$$=\frac{1}{64}\int_0^\pi \left(\frac{3}{2}-2\cos 4x+\frac{1}{2}\cos 8x\right)dx+\left(\frac{2}{5}-\frac{2}{7}\right)$$
$$=\frac{3}{128}[x]_0^\pi-\frac{2}{64}\left[\frac{\sin 4x}{4}\right]_0^\pi+\frac{1}{128}\left[\frac{\sin 8x}{8}\right]_0^\pi+\frac{4}{35}$$
$$\Rightarrow \frac{3\pi}{128}+\frac{4}{35}$$

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# Question6

$$\int_0^1 \frac{x^4+1}{x^6+1}dx=$$

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### Options:

A.

$\frac{\pi}{3}$

B.

$\frac{\pi}{4}$

C.

$\frac{\pi}{6}$

D.

$\frac{\pi}{2}$

**Answer: A**

## Solution:

$$I=\int_0^1 \frac{x^4+1}{x^6+1} dx$$
put $x^6+1=\left(x^2\right)^3+1$
$$=\left(x^2+1\right)\left(x^4-x^2+1\right)$$
$$=\int_0^1 \frac{\left(x^4+1\right)}{\left(x^2+1\right)\left(x^4-x^2+1\right)} dx$$
$$\Rightarrow \frac{x^4+1}{\left(x^2+1\right)\left(x^4-x^2+1\right)}$$
$$=\frac{A}{x^2+1}+\frac{Bx^2+C}{x^4-x^2+1}$$
$$x^4+1=A\left(x^4-x^2+1\right)+Bx^2\left(x^2+1\right)+Cx^2+C$$

| | | |
|---|---|---|
| $1=A+B$ | $-A+B+C=0$ | $A+C=1$ |
| $B=1-A$ | $A-B-C=0$ | $C=1-A$ |

$$\therefore \quad A-(1-A)-(1-A)=0$$
$$\Rightarrow \quad A-1+A-1+A=0$$
$$\Rightarrow \quad 3A=2$$
$$\Rightarrow \quad A=2/3$$
$$B=C=1-2/3=1/3$$
$$I=\frac{2}{3}\int_0^1 \frac{1}{x^2+1} dx-\frac{1}{3}\int \frac{x^2+1}{x^4-x^2+1} dx$$
$$=\frac{2}{3}\left[\tan^{-1} x\right]_0^1-\frac{1}{3}\int_0^1 \frac{1+1/x^2}{x^2+1/x^2-1} dx$$
$$=\frac{2}{3}\left(\frac{\pi}{4}\right)-\frac{1}{3}\int_0^1 \frac{\left(1+1/x^2\right)}{\left(x-\frac{1}{x}\right)^2+1} dx$$
$$=\frac{\pi}{6}-\frac{1}{3}\left[\tan^{-1}(x-1/x)\right]_0^1$$
$$=\frac{\pi}{6}-1/3\left(\frac{-\pi}{2}\right)$$
$$\Rightarrow \frac{\pi}{6}+\pi/6=\frac{\pi}{3}$$

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# Question7

$\int_{-2\pi}^{2\pi} \sin^4(2x)\cos^6(2x)dx =$

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**Options:**

A.

$\frac{3\pi}{64}$

B.

$\frac{9\pi}{64}$

C.

$\frac{9\pi}{35}$

D.

$\frac{9\pi}{280}$

**Answer: A**

## Solution:

$I=\int_{-2\pi}^{2\pi} \sin^4(2x)\cos^6(2x)dx$

Let $u=2x$, then $du=2dx$,

so $dx=\frac{1}{2}du$

When $x=-2\pi, u=2(-2\pi)=-4\pi$

when $x=2\pi, u=2(2\pi)=4\pi$

$$\begin{aligned} I &= \int_{-2\pi}^{2\pi} \sin^4(2x)\cos^6(2x)dx \\ &= \frac{1}{2}\int_{-4\pi}^{4\pi} \sin^4(u)\cos^6(u)du \end{aligned}$$

Since, $\sin(u)$ and $\cos(u)$ are periodic with a period of $2\pi$, So, $\sin^4(u)$ and $\cos^6(u)$ are also periodic with a period of $2\pi$ $\quad\ldots(ii)$

$$= \frac{1}{2} \cdot 2 \int_{-2\pi}^{2\pi} \sin^4(u) \cos^6(u) du$$
$$= \int_{-2\pi}^{2\pi} \sin^4(u) \cos^6(u) du$$
$$= \frac{1}{2} \cdot 4 \int_{0}^{2\pi} \sin^4(u) \cos^6(u) du$$
$$= 2 \int_{0}^{2\pi} \sin^4(u) \cos^6(u) du$$

Since, the function, $f(u) = \sin^4(u) \cos^6(u)$ is an even function.

$$\therefore 2 \int_{0}^{2\pi} \sin^4(u) \cos^6(u) du$$
$$= 2 \cdot 2 \int_{0}^{\pi} \sin^4(u) \cos^6(u) du$$
$$= 4 \int_{0}^{\pi} \sin^4(u) \cos^6(u) du$$
$$= 4 \cdot 2 \int_{0}^{\pi/2} \sin^4(u) \cos^6(u) du$$
$$= 8 \int_{0}^{\pi/2} \sin^4(u) \cos^6(u) du$$

We know that, $\int_0^{\pi/2} \sin^m(u) \cos^n(u) du$

$$= \frac{(m-1)(m-3)\ldots(1 \text{ or } 2)(n-1)(n-3)\ldots(1 \text{ or } 2)}{(m+n)(m+n-2)\ldots(1 \text{ or } 2)} \times \alpha$$

where, $\alpha = \frac{\pi}{2}$ if both $m$ and $n$ are even, and $\alpha = 1$ othernels.

Here, $m = 4, n = 6$ both are even.

So, $\int_0^{\pi/2} \sin^4(4) \cos^6(4) du$

$$= \frac{(4-1)(4-3)(6-1)(6-3)(6-5)}{(4+6)(4+6-2)(4+6-4)} \cdot \frac{\pi}{2}$$
$$(4+6-6)(4+6-8)$$
$$= \frac{3 \cdot 1 \cdot 5 \cdot 3 \cdot 1}{10 \cdot 8 \cdot 6 \cdot 4 \cdot 2} \cdot \frac{\pi}{2} = \frac{45}{3840} \cdot \frac{\pi}{2} = \frac{3}{256} \cdot \frac{\pi}{2}$$
$$= \frac{3\pi}{512}$$
$$\therefore 8 \int_0^{\pi/2} \sin^4(u) \cos^6(u) du$$
$$= 8 \cdot \frac{3\pi}{512} = \frac{24\pi}{512} = \frac{3\pi}{64}$$
$$\therefore 8 \int_0^{\frac{\pi}{2}} \sin^4(u) \cos^6(u) du = \frac{3\pi}{64}$$

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# Question8

**If $f(t) = \int_0^t \tan^{(2n-1)} x dx, n \in N$, then $f(t+\pi) =$**

**Options:**

A.

$f(t) f(\pi)$

B.

$f(t)-f(\pi)$

C.

$f(t)+f(\pi)$

D.

$\frac{f(t)}{f(\pi)}$

**Answer: C**

## Solution:

Given, $f(t)=\int_0^t \tan ^{(2 n-1)} x d x, n \in$

So, $f(t+\pi)=\int_0^{t+\pi} \tan ^{(2 n-1)} x d x$

$=\int_0^t \tan ^{(2 n-1)} x d x+\int_t^{t+\pi} \tan ^{(2 n-1)} x d x$

Let $u=x-\pi$

$\Rightarrow d u=d x$

Limit, when $x=t, u=t-\pi$ and

$x=t+\pi, u=t$

So, $f(t+\pi)=f(t)+\int_t^t \tan ^{(2 n-1)}(u+\pi) d u$

Since, $\tan (u+\pi)=\tan u$

$=f(t)+\int_{t-\pi}^t \tan ^{(2 n-1}(u) d u$

$=f(t)+\int_{t-\pi}^0 \tan ^{(2 n-1)}(u) d u+\int_0^t \tan ^{(2 n-1)}(u) d u$

Let $v=-u$, the $d v=-d u$

When $u=t-\pi, v=\pi-t$, when $u=0, v=0$, so we get

$$= f(t) - \int_{\pi-t}^{0} \tan^{(2n-1)}(-v)dv + \int_0^t \tan^{(2n-1)}(u)du$$

$$= f(t) + \int_{\pi-t}^{0} \tan^{(2n-1)}(v)dv + \int_0^t \tan^{(2n-1)}(u)du$$

$$= f(t) + \int_{\pi-t}^{0} \tan^{(2n-1)}(v)dv + f(t)$$

$$= 2f(t) - \int_0^{\pi-t} \tan^{(2n-1)}(v)dv$$

$$= 2f(t) - \int_0^{\pi} \tan^{(2n-1)}(v)dv - \int_{\pi-t}^{\pi} \tan^{(2n-1)}(v)dv$$

Let $w = v - \pi$, then $dw = dv$

When $v = \pi - t, w = -t,$ when $v = \pi$

$w = 0$

$$= 2f(t) - \int_0^{\pi} \tan^{(2n-1)}(v)dv - \int_{-t}^{0} \tan^{(2n-1)}(w+\pi)dw$$

$$2f(t) - \int_0^{\pi} \tan^{(2n-1)}(v)dv - \int_{-t}^{0} \tan^{(2n-1)}(w)dw$$

Let $z = -w$, then $dz = -dw$ when $w = -t$, $z = t$ and $w = 0, z = 0$

$$=2f(t) - \int_0^{\pi} \tan^{(2n-1)}(v)dv + \int_t^{0} \tan^{(2n-1)}(-z)dz$$

$$=2f(t) - \int_0^{\pi} \tan^{(2n-1)}(v)dv - \int_0^{t} \tan^{(2n-1)}(z)dz$$

$$=2f(t) - \int_0^{\pi} \tan^{(2n-1)}(v)dv - f(t)$$

$$=f(t) - \int_0^{\pi} \tan^{(2n-1)}(v)dv$$

$$=f(t) + 0 \left[\because \int_0^{\pi} \tan^{(2n-1)}(v)dv = 0\right]$$

$$= f(t) + f(\pi) \quad [\because f(\pi) = 0]$$

$$\therefore \quad f(t+\pi) = f(t) + f(\pi)$$

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# Question9

$$\int_0^2 x^8\left(\frac{4}{x^2} - 1\right)^{\frac{5}{2}} dx =$$

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**Options:**

A.

$\frac{2^{15}}{63}$

B.

$\frac{2^{16}}{315}$

C.

$\frac{2^{16}}{189}$

D.

$\frac{2^{10}}{63}$

**Answer: D**

## Solution:

$$\begin{aligned} I &= \int_0^2 x^8 \left(\frac{4}{x^2} - 1\right)^{5/2} dx \\ &= \int_0^2 x^8 \left(\frac{4 - x^2}{x^2}\right)^{5/2} dx \\ &= \int_0^2 x^8 \cdot \frac{\left(4 - x^2\right)^{5/2}}{x^5} dx \\ &= \int_0^2 x^3 \left(4 - x^2\right)^{5/2} dx \end{aligned}$$

Let $u = 4 - x^2$

Then, $du = -2x dx$, so $x dx = \frac{-1}{2} du x^2 = 4 - u$ and $x^3 = x \cdot x^2 = x(4 - u)$ when $x = 0, u = 4$ and when $x = 2, u = 0$

$$\therefore \int_0^2 x^8\left(\frac{4}{x^2}-1\right)^{5/2} dx$$

$$= \int_0^2 x^3\left(4-x^2\right)^{5/2} dx$$

$$= \int_4^0 (4-u)\cdot u^{5/2}\cdot\left(\frac{-1}{2}\right) du$$

$$= \frac{1}{2}\left[4\cdot\frac{u^{7/2}}{7/2}-\frac{u^{9/2}}{9/2}\right]_0^4$$

$$= \frac{1}{2}\left[\frac{8}{7}u^{7/2}-\frac{2}{9}u^{9/2}\right]_0^4$$

$$= \frac{1}{2}\left[\frac{8}{7}(u)^{7/2}-\frac{2}{9}(4)^{9/2}-0\right]$$

$$= \frac{1}{2}\left[\frac{8}{7}(128)-\frac{2}{9}(512)\right]$$

$$= \frac{1}{2}\left[\frac{1024}{7}-\frac{1024}{9}\right]$$

$$= \frac{1024}{2}\cdot\frac{2}{63} = \frac{1024}{63}$$

$$\therefore \int_0^2 x^8\left(\frac{4}{x^2}-1\right)^{5/2} dx = \frac{1024}{63} = \frac{2^{10}}{63}$$

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# Question10

$\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x(\sin x+\cos x) dx =$

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**Options:**

A.

0

B.

$\frac{2}{15}$

C.

$\frac{4}{15}$

D.

$\frac{2}{5}$

**Answer: C**

## Solution:

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cdot \cos^2 x(\sin x+\cos x) dx$$
$$=\int_{-\pi 2}^{\pi / 2} \sin^3 x \cos^2 x dx$$
$$+\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^3 x dx$$
$$=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin x \cdot\left(1-\cos^2 x\right) \cos^2 x dx$$
$$+\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos x\left(1-\sin^2 x\right) \cdot \sin^2 x dx$$
$$=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin x \cdot \cos^2 x \cdot dx$$
$$-\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin x \cdot \cos^4 x dx$$
$$+\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos x \cdot \sin^2 x \cdot dx-\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos x \cdot \sin^4 x dx$$
$$=\left[\frac{\cos^3 x}{3}+\frac{\cos^5 x}{5}+\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}\right]_{-\frac{\pi}{2}}^{\pi}$$
$$=\left(0+0+\frac{1}{3}-\frac{1}{5}\right)-\left(0+0-\frac{1}{3}+\frac{1}{5}\right)$$
$$=\frac{2}{3}-\frac{2}{5}=\frac{10-6}{15}=\frac{4}{15}$$

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# Question11

$\int_{1/5}^{1/2} \frac{\sqrt{x-x^2}}{x^3} dx =$

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**Options:**

A.

$\frac{21}{2}$

B.

$\frac{14}{3}$

C.

$\frac{7}{3}$

D.

$\frac{7}{2}$

**Answer: B**

## Solution:

$\int_{\frac{1}{5}}^{\frac{1}{2}} \frac{\sqrt{x-x^2}}{x^3} dx$

$= \int_{\frac{1}{5}}^{\frac{1}{2}} \frac{x\sqrt{\frac{1}{x}-1}}{x^3} dx \int_{\frac{1}{5}}^{\frac{1}{2}} \frac{1}{x^2} \sqrt{\frac{1}{x} - 1.dx}$

Put $\frac{1}{x} - 1 = t \Rightarrow \frac{1}{x^2} dx = -dt$

at $x = \frac{1}{5} \Rightarrow t = 4$ and at $x = \frac{1}{2} \Rightarrow t = 1$

$$= -\int_4^1 t^{\frac{1}{2}} \cdot dt = \int_1^4 \sqrt{t} \cdot dt$$
$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 = \frac{2}{3}\left[t^{\frac{3}{2}}\right]_1^4$$
$$= \frac{2}{3}\left[4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] = \frac{2}{3}(8-1)$$
$$= \frac{14}{3}$$

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# Question12

$\int_0^{400\pi} \sqrt{1 - \cos 2x} dx =$

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**Options:**

A.

$100\sqrt{2}$

B.

$200\sqrt{2}$

C.

$400\sqrt{2}$

D.

$800\sqrt{2}$

**Answer: D**

## Solution:

$$
\begin{aligned}
I &= \int_0^{400\pi} \sqrt{1-\cos 2x}\, dx \\
&= \int_0^{400\pi} \sqrt{2\sin^2 x}\, dx \\
&= \sqrt{2}\int_0^{400\pi} \sin x\, dx \\
&= 400\sqrt{2}\int_0^{\pi} \sin x\, dx \\
&= 2\times 400\sqrt{2}\int_0^{\pi/2} \sin x\, dx \\
&= 800\sqrt{2}[-\cos x]_0^{\frac{\pi}{2}} \\
&= 800\sqrt{2}\left[-\cos\frac{\pi}{2}+\cos 0\right] \\
&= 800\sqrt{2}
\end{aligned}
$$

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# Question13

$$\int_0^x \frac{t^2}{\sqrt{a^2+t^2}} dt =$$

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**Options:**

A.

$\frac{x}{2}\sqrt{a^2+x^2}+\log\left|x+\sqrt{a^2+x^2}\right|$

B.

$\sqrt{a^2+x^2}-a^2\sinh^{-1}\frac{x}{a}$

C.

$\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{4}\log\left|x+\sqrt{a^2+x^2}\right|$

D.

$\frac{x}{2}\sqrt{a^2+x^2}-\frac{a^2}{2}\sinh^{-1}\frac{x}{a}$

**Answer: D**

## Solution:

Let $I=\int_0^x \frac{t^2}{\sqrt{a^2+t^2}}dt$

Put $t=a\sin h\theta \Rightarrow dt=a\cos h\theta d\theta$

Now, $\sqrt{a^2+t^2}=\sqrt{a^2+a^2\sin h^2\theta}$

$=a\sqrt{1+\sinh^2\theta}=a\cos h\theta$

and $\quad t^2=a^2\sin h^2\theta$

| $t$ | $0$ | $x$ |
|---|---|---|
| $\theta$ | $0$ | $\sin h^{-1}\left(\frac{x}{4}\right)$ |

$$\therefore I=\int_0^{\sin h^{-1}\left(\frac{x}{a}\right)}\frac{a^2\sin h^2\theta}{a\cos h\theta}\cdot a\cos h\theta d\theta$$

$$=\int_0^{\sin h^{-1}\left(\frac{x}{a}\right)}a^2\sin h^2\theta d\theta$$

$$\because \sin h^2\theta=\frac{\cos h(2\theta)-1}{2}$$

So, $I = a^2 \int_0^{\sin h^{-1}\left(\frac{x}{a}\right)} \frac{\cos h(2\theta) - 1}{2} d\theta = \frac{a^2}{2}$

$\left[\int_0^{\cos h(2\theta) d\theta - \int_0^{\sin h^{-1}\left(\frac{x}{a}\right)}} {}^{\sin h^{-1}\left(\frac{x}{a}\right)} \int_1 1 \cdot d\theta\right]$

$= \frac{a^2}{2}\left[\frac{1}{2}\sin h(2\theta) - \theta\right] = \frac{a^2}{2}$

$= \frac{a^2}{2}\left[\frac{x}{a} \times \frac{\sqrt{a^2+x^2}}{a} - \sin h^{-1}\left(\frac{x}{a}\right)\right]$

$= \frac{x}{2}\sqrt{a^2+x^2} - \frac{a^2}{2}\sin h^{-1}\left(\frac{x}{a}\right)$

---

# Question14

$\int_{\frac{5}{6}}^{\pi} \cos^{-4} x dx =$

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**Options:**

A.

$\frac{64}{9\sqrt{3}}$

B.

$\frac{52\sqrt{3}}{9}$

C.

$\frac{62\sqrt{3}}{9}$

D.

$\frac{44}{9\sqrt{3}}$

**Answer: D**

## Solution:

Let $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^4 x dx$

$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\cos^4 x} dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^4 x dx$$

$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x\left(1+\tan^2 x\right) dx=\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}\left(1+u^2\right) du$$

| $x$ | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ |
|---|---|---|
| $u$ | $\frac{1}{\sqrt{3}}$ | $\sqrt{3}$ |

where $u=\tan x$

$$=\left(u+\frac{u^3}{3}\right)_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$$

$$=\left(\sqrt{3}+\frac{(\sqrt{3})^3}{3}\right)-\left(\frac{1}{\sqrt{3}}+\frac{\left(\frac{1}{\sqrt{3}}\right)^3}{3}\right)$$

$$=\frac{3\sqrt{3}+3\sqrt{3}}{3}-\left(\frac{1}{\sqrt{3}}+\frac{1}{9\sqrt{3}}\right)$$

$$=\frac{6\sqrt{3}}{3}-\left(\frac{9+1}{9\sqrt{3}}\right)=2\sqrt{3}-\frac{10\sqrt{3}}{27}$$

$$=\frac{54\sqrt{3}-10\sqrt{3}}{27}=\frac{44\sqrt{3}}{27}=\frac{44}{9\sqrt{3}}$$

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# Question15

$$\int_0^{\frac{3\pi}{2}} \frac{\cos^3 x}{\cos^3 x+\sin^3 x} dx=$$

## AP EAPCET 2025 - 23rd May Morning Shift

**Options:**

A.

0

B.

1

C.

$\frac{\pi}{4}$

D.

$\frac{3\pi}{4}$

**Answer: D**

## Solution:

$$Let\ I=\int_0^{\frac{3\pi}{2}} \frac{\cos^3 x}{\cos^3 x+\sin^3 x} dx \qquad \ldots(i)$$

$$I=\int_0^{\frac{3\pi}{2}} \frac{\left(\cos\left(\frac{3\pi}{2}-x\right)\right)^3}{\left(\cos\left(\frac{3\pi}{2}-x\right)\right)^3+\left(\sin\left(\frac{3\pi}{2}-x\right)\right)^3} dx$$

$$=\int_0^{\frac{3\pi}{2}} \frac{-\sin^3 x}{-\sin^3 x-\cos^3 x} dx$$

$$=\int_0^{\frac{3\pi}{2}} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx \qquad \ldots(ii)$$

On adding in Eqs. (i) and (ii), we get

$\Rightarrow \quad 2I=\int_0^{\frac{3\pi}{2}} dx=\frac{3\pi}{2}$

$\therefore \quad I=\frac{3\pi}{4}$

---

# Question16

**If $k \in N$, then $\lim\limits_{n \rightarrow \infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\ldots .+\frac{1}{kn}\right]=$**

## AP EAPCET 2025 - 22nd May Evening Shift

**Options:**

A.

$\log(k+1)$

B.

$\log k$

C.

$\log(k+5)$

D.

$\log(k+1)-\log 6$

**Answer: B**

## Solution:

$$\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\ldots+\frac{1}{kn}\right]$$
$$\sum_{i=1}^{(k-1)n}\frac{1}{n+i}$$
$$\Rightarrow \frac{1}{n}\sum_{i=1}^{(k-1)n}\frac{1}{1+\frac{i}{n}}=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{(k-1)n}\frac{1}{1+\frac{i}{n}}$$
$$=\int_0^{k-1}\frac{1}{1+x}dx=[\ln(1+x)]_0^{k-1}$$
$$=\ln k$$

----------------------------------------------------------------------------------------------------

# Question17

$$\int_{-1}^{4}\sqrt{\frac{4-x}{x+1}}dx=$$

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**Options:**

A.

$0$

B.

$\frac{\pi}{2}$

C.

$\frac{3\pi}{2}$

D.

$\frac{5\pi}{2}$

**Answer: D**

## Solution:

$$\int_{-1}^{4}\sqrt{\frac{4-x}{x+1}}dx$$

$$\text{Let } x=\frac{5}{2}\sin\theta+\frac{3}{2}$$

$$\Rightarrow dx=\frac{5}{2}\cos\theta d\theta$$

$$\Rightarrow 4-x=4-\left(\frac{5}{2}\sin\theta+\frac{3}{2}\right)$$

$$=\frac{5}{2}(1-\sin\theta)$$

$$\Rightarrow x+1=\frac{5}{2}\sin\theta+\frac{3}{2}+1=\frac{5}{2}(1+\sin\theta)$$

$$\Rightarrow \quad \frac{4-x}{x+1}=\frac{1-\sin\theta}{1+\sin\theta}$$

$$\Rightarrow \quad \sqrt{\frac{4-x}{x+1}}=\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\frac{1-\sin\theta}{\cos\theta}$$

$$\int\sqrt{\frac{4-x}{x+1}}dx=\int\frac{1-\sin\theta}{\cos\theta}\frac{5}{2}\cos\theta d\theta$$

$$=\frac{5}{2}\int(1-\sin\theta)d\theta$$

$$=\frac{5}{2}(\theta+\cos\theta)+c$$

$$x=\frac{5}{2}\sin\theta+\frac{3}{2}\Rightarrow\sin\theta=\frac{2x-3}{5}$$

$$\Rightarrow\cos\theta=\sqrt{1-\sin^2\theta}=\frac{2}{5}\sqrt{4+3x-x^2}$$

$$\int_{-1}^{4}\sqrt{\frac{4-x}{x+1}}dx=\left[\frac{5}{2}\sin^{-1}\left(\frac{2x-3}{5}\right)+\sqrt{4+3x-x^2}\right]_{-1}^{4}$$

$$=\frac{5}{2}\cdot\frac{\pi}{2}-\frac{5}{2}\left(\frac{-\pi}{2}\right)$$

$$=\frac{5\pi}{4}+\frac{5\pi}{4}=\frac{5\pi}{2}$$

---

# Question18

$$\int_0^{\pi/4}\frac{\cos^2 x}{\cos^2 x+4\sin^2 x}dx=$$

**Options:**

A.

$\frac{\pi}{2}-\frac{1}{3}\tan^{-1}2$

B.

$-\frac{\pi}{4}-\frac{4}{3}\tan^{-1}2$

C.

$\frac{\pi}{6}+\frac{2}{3}\tan^{-1}2$

D.

$-\frac{\pi}{12}+\frac{2}{3}\tan^{-1}2$

**Answer: D**

## Solution:

$$\int_0^{\frac{\pi}{4}}\frac{\cos^2 x dx}{\cos^2 x+4\sin^2 x}$$
$$=\int_0^{\frac{\pi}{4}}\frac{1}{1+4\tan^2 x}dx$$

Put $u=2\tan x$

$$du=2\sec^2 x dx$$
$$dx=\frac{du}{2\left(1+\tan^2 x\right)}=\frac{2du}{4+u^2}$$
$$x=0, u=0$$
$$x=\frac{\pi}{4}, u=2$$
$$=\int_0^2\frac{1}{1+u^2}\cdot\frac{2du}{4+u^2}$$
$$=\int_0^2\left(\frac{2}{3}\frac{1}{1+u^2}-\frac{2}{3}\frac{1}{4+u^2}\right)du$$
$$=\frac{2}{3}\left[\tan^{-1}u\right]_0^2-\frac{2}{3}\cdot\frac{1}{2}\left[\tan^{-1}\frac{u}{2}\right]_0^2$$
$$=\frac{2}{3}\tan^{-1}(2)-\frac{\pi}{12}$$

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# Question19

$\int_{5\pi}^{25\pi} |\sin 2x + \cos 2x| dx =$

## AP EAPCET 2025 - 22nd May Evening Shift

**Options:**

A.

$20\sqrt{2}$

B.

$10\sqrt{2}$

C.

$40\sqrt{2}$

D.

$80\sqrt{2}$

**Answer: C**

## Solution:

$$\int_{5\pi}^{25\pi} \sin 2x + \cos 2x \mid dx$$

$$\Rightarrow |\sin 2x + \cos 2x| = \sqrt{2}\left|\sin\left(2x + \frac{\pi}{4}\right)\right|$$

$$\Rightarrow \int_{5\pi}^{25\pi} \sin 2x + \cos 2x \mid dx$$

$$= \int_{5\pi}^{25\pi} \sqrt{2}\left|\sin\left(2x + \frac{\pi}{4}\right)\right| dx$$

Let $u = 2x + \frac{\pi}{4}$

$du = 2dx$

$dx = \frac{1}{2}du$

$x = 5\pi, u = 10\pi + \frac{\pi}{4}$

$x = 25\pi, u = 50\pi + \frac{\pi}{4}$

$= \sqrt{2}\int_{10\pi+\frac{\pi}{4}}^{50\pi+\frac{\pi}{4}} |\sin u| \frac{1}{2}du$

$= \frac{\sqrt{2}}{2}\int_{10\pi+\frac{\pi}{4}}^{50\pi+\frac{\pi}{4}} |\sin u| du$

The period of $|\sin u|$ is $\pi$.

The length of the interval of integration is

$= \left(50\pi + \frac{\pi}{4}\right) - \left(10\pi + \frac{\pi}{4}\right) = 40\pi$

$= \frac{\sqrt{2}}{2} \cdot 40 \int_0^{\pi} |\sin u| du$

$= \frac{\sqrt{2}}{2} \times 40 \times 2$

$= 40\sqrt{2}$

---

# Question20

$\int_{\frac{-\pi}{4}}^{\frac{\pi}{3}} \left|\tan\left(x - \frac{\pi}{6}\right)\right| dx =$

## AP EAPCET 2025 - 22nd May Morning Shift

**Options:**

A.

$\log \frac{\sqrt{3}-1}{\sqrt{6}}$

B.

$\log(2\sqrt{2}(\sqrt{3}+1))$

C.

$\log \frac{\sqrt{3}+1}{\sqrt{6}}$

D.

$\log(2\sqrt{2}(\sqrt{3}-1))$

**Answer: B**

## Solution:

$I=\int_{\frac{-\pi}{4}}^{\frac{\pi}{3}}\left|\tan\left(x-\frac{\pi}{6}\right)\right|dx$

Put $\tan\left(x-\frac{\pi}{6}\right)=0 \Rightarrow x-\frac{\pi}{6}=0$

$$I=\int_{\frac{-\pi}{4}}^{\frac{\pi}{6}}-\tan\left(x-\frac{\pi}{6}\right)dx+\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\tan\left(x-\frac{\pi}{6}\right)dx$$

$$=-\left[\log\left(\sec\left(x-\frac{\pi}{6}\right)\right]_{-\pi}^{\frac{\pi}{6}}+\left[\log\sec\left(x-\frac{\pi}{6}\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}\right.$$

$$\Rightarrow\log\left(\sec\left(\frac{5\pi}{12}\right)\right)+\log\left(\sec\left(\frac{\pi}{6}\right)\right)$$

$$=\log\left(\sec\left(\frac{5\pi}{12}\right)\sec\left(\frac{\pi}{6}\right)\right)$$

$$=\log\left(2\sqrt{2}\frac{(\sqrt{3}+1)}{\sqrt{3}}\right)$$

---

# Question21

$$\int_0^\pi\frac{x\sin x}{\sin^2x+2\cos^2x}dx=$$

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**Options:**

A.

$\frac{\pi}{2}$

B.

$\frac{\pi^2}{2}$

C.

$\frac{\pi^2}{4}$

D.

$\frac{\pi}{4}$

**Answer: C**

## Solution:

$$I=\int_0^\pi \frac{x \sin x}{\sin ^2 x+2 \cos ^2 x} d x \quad \ldots(i)$$
$$\Rightarrow I=\int_0^\pi \frac{(\pi-x) \sin x}{\sin ^2 x+2 \cos ^2 x} d x$$
$$\left[\because \int_0^a f(x) d x=\int_0^a f(a-x) d x\right]$$

$$\Rightarrow 2 I=\int_0^\pi \frac{\pi \sin x}{1+\cos ^2 x} d x$$
Put $\cos x=t \Rightarrow-\sin x d x=d t$
$$\Rightarrow 2 I=-\pi\left[\tan ^{-1} t\right]_1^{-1}$$
$$\Rightarrow \quad 2 I=-\pi\left[\frac{-\pi}{4}-\frac{\pi}{4}\right] \Rightarrow I=\frac{\pi^2}{4}$$

---

# Question22

$$\lim _{n \rightarrow \infty}\left(\frac{1}{1^2+n^2}+\frac{2}{2^2+n^2}+\frac{3}{3^2+n^2}+\ldots+\frac{n}{n^2+n^2}\right)=$$

## AP EAPCET 2025 - 22nd May Morning Shift

**Options:**

A.

1

B.

$\frac{1}{2} \log 2$

C.

$2 \log 2$

D.

0

**Answer: B**

## Solution:

$$\lim_{n\to\infty}\left[\frac{1}{1^2+n^2}+\frac{2}{2^2+n^2}+\frac{3}{3^2+n^2}+\frac{n}{n^2+n^2}\right]$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{r^2+n^2}=\lim_{n\to\infty}\sum\frac{\left(\frac{r}{n}\right)}{1+\left(\frac{r}{n}\right)^2}\cdot\frac{1}{n}$$

$$\Rightarrow\int_0^1\frac{x}{1+x^2}dx$$

put $1+x^2=t\Rightarrow 2xdx=dt$

$$\Rightarrow\frac{1}{2}\int_1^2\frac{1}{t}dt\Rightarrow\frac{1}{2}[\log t]_1^2=\frac{1}{2}\log 2$$

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# Question23

$\int_0^{\frac{\pi}{2}}\log|\tan x+\cot x|dx=$

## AP EAPCET 2025 - 21st May Evening Shift

**Options:**

A.

$\pi\log 2$

B.

$-\pi\log 2$

C.

$\frac{\pi}{2}\log 2$

D.

$2\pi\log 2$

**Answer: A**

## Solution:

$$\int_0^{\frac{\pi}{2}} \ln(\tan x + \cot x)dx = \int_0^{\frac{\pi}{2}} \ln\left(\frac{1}{\sin x \cos x}\right)dx$$

$$= \int_0^{\frac{\pi}{2}} (\ln 2 - \ln \sin 2x)dx$$

$$= \ln 2 \int_0^{\frac{\pi}{2}} dx - \int_0^{\frac{\pi}{2}} \ln \sin 2x dx$$

$$\text{put } 2x = t, dx = \frac{dt}{2}$$

$$= \frac{\pi}{2}\ln 2 - \frac{1}{2}\int_0^{\pi} \ln \sin t dt$$

$$= \frac{\pi}{2}\ln 2 - \int_0^{\pi/2} \ln \sin t dt$$

$$= \frac{\pi}{2}\ln 2 - \left(-\frac{\pi}{2}\ln 2\right)$$

$$\left[\because \int_0^{\pi/2} \ln \sin x = -\tfrac{\pi}{2}\ln 2\right]$$

$$= \pi \ln 2$$

---

# Question24

$\int_0^{\pi} x \cdot \sin^5 x \cdot \cos^6 x dx =$

### AP EAPCET 2025 - 21st May Evening Shift

**Options:**

A.

$\frac{16\pi}{693}$

B.

$\frac{8\pi}{693}$

C.

$\frac{4\pi}{693}$

D.

$\frac{2\pi}{693}$

**Answer: B**

## Solution:

Let $I=\int_0^\pi x\sin^5 x\cos^6 x dx$

Using $\int_a^b xf(x)dx=\frac{a+b}{2}\int_a^b f(x)dx$
If $f(a+b-x)=f(x)$

$$\Rightarrow \quad I=\frac{\pi}{2}\int_0^\pi \sin^5 x\cos^6 x dx$$
$$=\frac{\pi}{2}2\int_0^{\pi/2}\sin^5 x\cos^6 x dx$$

By Walli's Formula,
$$=\pi\left[\frac{(4\cdot 2)(5\cdot 3\cdot 1)}{11\cdot 9\cdot 7\cdot 5\cdot 3\cdot 1}\right]=\frac{8\pi}{693}$$

------------------------------------------------------------------------------------------------

# Question25

$$\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}\frac{1}{\left(x+\sqrt{1-x^2}\right)(1-x^2)}dx=$$

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**Options:**

A.

$\log(\sqrt{3}+1)$

B.

$\log(\sqrt{3}-1)$

C.

$\log(3+\sqrt{3})$

D.

$\log(3-\sqrt{3})$

**Answer: D**

## Solution:

$$I=\int_{1/2}^{1/\sqrt{2}} \frac{1}{\left(x+\sqrt{1-x^2}\right)\left(1-x^2\right)} dx$$

$$I=\int_{\pi/6}^{\pi/4} \frac{\cos\theta d\theta}{(\sin\theta+\cos\theta)\cos^2\theta}$$

Put $x=\sin\theta \Rightarrow dx=\cos\theta d\theta$

$$=\int_{\pi/6}^{\pi/4} \frac{d\theta}{\sin\theta\cos\theta+\cos^2\theta}$$

Dividing $N^T$ and $D^r$ by $\cos^2\theta$, we get

$=\int_{\pi/6}^{\pi/4} \frac{\sec^2\theta}{\tan\theta+1} d\theta$

Put $\tan\theta+1=t, \sec^2\theta d\theta=dt$

$$=\int_{\frac{1}{\sqrt{3}}+1}^{2} \frac{dt}{t}=[\ln t]_{1+\frac{1}{\sqrt{3}}}^{2}$$

$$=\ln 2-\ln\left(1+\frac{1}{\sqrt{3}}\right)$$

$$=\ln 2-\ln\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)=\ln\left(\frac{2\sqrt{3}}{\sqrt{3}+1}\right)$$

$$=\ln\left(\frac{2\sqrt{3}(\sqrt{3}-1)}{3-1}\right)=\ln(3-\sqrt{3})$$

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# Question26

**Let $H(x)=3x^4+6x^3-2x^2+1$ and $g(x)$ be a linear polynomial. If $\frac{H(x)}{(x-1)(x+1)(x-2)}=f(x)+\frac{g(x)}{(x-1)(x+1)(x-2)}$, then**

$H(-1)+2H(2)-3H(1)=$

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**Options:**

A.

$f(-1)+2f(2)-3f(1)$

B.

$H(-1)+f(2)+g(3)$

C.

$g(-1)+2g(2)-3g(1)$

D.

$H(1)+2f(2)-g(1)$

**Answer: C**

## Solution:

$$\begin{aligned}
&\text{Given, } H(x)=3x^4+6x^3-2x^2+1\\
&\text{Given, } \frac{H(x)}{(x-1)(x-2)(x+1)}\\
&=f(x)+\frac{g(x)}{(x-1)(x+1)(x-2} \text{ taking LCM}\\
&H(x)=f(x)(x-1)(x+1)(x-2)+g(x)\\
&H(-1)=g(-1)\\
&H(2)=g(2)\\
&H(1)=g(1)\\
&\text{Now, } H(-1)+2H(2)-3H(1)\\
&=g(-1)+2g(2-3g(1)
\end{aligned}$$

---

# Question27

$\int_{\pi/4}^{\pi/3} \frac{\cos x-\sin x}{\sin 2x} dx =$

### AP EAPCET 2025 - 21st May Morning Shift

**Options:**

A.

$\frac{1}{2}\log\left[\frac{(3+2\sqrt{2})(2-\sqrt{3})}{\sqrt{3}}\right]$

B.

$\frac{1}{2}\log\left[\frac{(3-2\sqrt{2})(2+\sqrt{3})}{\sqrt{3}}\right]$

C.

$\log\left[\frac{(3-2\sqrt{2})(2-\sqrt{3})}{\sqrt{3}}\right]$

D.

$\log\left[\frac{(3+2\sqrt{2})(2-\sqrt{3})}{\sqrt{3}}\right]$

**Answer: A**

## Solution:

$$I=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\cos x-\sin x}{\sin 2x}dx$$

$$I=\int_{\sqrt{2}}^{\frac{\sqrt{3}}{2}+\frac{1}{2}}\frac{dt}{t^2-1}$$

Put $\sin x+\cos x=t$

$(\cos x-\sin x)dx=dt$

$1+\sin 2x=t^2$

$$=\left[\frac{1}{2}\ln\left(\frac{t-1}{t+1}\right)\right]_{\sqrt{2}}^{\frac{\sqrt{3}+1}{2}}$$

$$=\frac{1}{2}\left[\ln\left(\frac{\frac{\sqrt{3}+1}{2}-1}{\frac{\sqrt{3}+1}{2}+1}\right)-\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right]$$

$$=\frac{1}{2}\left[\ln\frac{\sqrt{3}-1}{\sqrt{3}+3}-\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right]$$

$$=\frac{1}{2}\left[\ln\frac{2(2\sqrt{3}-3)}{6}-\ln(3-2\sqrt{2})\right]$$

$$=\frac{1}{2}\left[\ln\frac{2\sqrt{3}-3}{3}+\ln(3+2\sqrt{2})\right]$$

$$=\frac{1}{2}\left[\ln\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right)+\ln(3+2\sqrt{2})\right]$$

$$=\frac{1}{2}\ln\left[\frac{(3+2\sqrt{2})(2-\sqrt{3})}{\sqrt{3}}\right]$$

---

# Question28

$\int_0^{\pi/2} \frac{\sin x}{1+\cos x+\sin x} dx =$

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**Options:**

A.

$\frac{\pi}{2} + \frac{1}{2}\log 2$

B.

$\frac{\pi}{4} - \frac{1}{2}\log 2$

C.

$\frac{\pi}{4}$

D.

$\frac{3\pi}{4} + \log 2$

**Answer: B**

## Solution:

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x+\sin x} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}} dx \end{aligned}$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\sin \frac{x}{2} + \cos \frac{x}{2} + \sin \frac{x}{2} - \cos \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}} dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \left(1 + \frac{\sin \frac{x}{2} - \cos \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}}\right) dx$$

$$= \frac{1}{2} \left[x + \left\{-2 \ln \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)\right\}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2} \left[x - 2 \ln \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2} \left[\left(\frac{\pi}{2} - 2 \ln \sqrt{2}\right) - 0\right]$$

$$= \frac{\pi}{4} - \ln \sqrt{2} = \frac{\pi}{4} - \frac{1}{2} \ln 2$$

---

# Question29

$\int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx =$

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**Options:**

A.

$\frac{\pi^2}{4}$

B.

$\frac{\pi}{2}$

C.

$\frac{\pi^2}{2}$

D.

$\frac{\pi}{4}$

**Answer: A**

## Solution:

$I = \int_0^\pi \frac{x \sin x}{1+\cos^2 x} \qquad \ldots (i)$

Apply property

$$\int_a^b x f(x) dx = \frac{a+b}{2} \int f(x) dx$$
If $f(a+b-x) = f(x)$

$$I = \frac{\pi}{2} \int_0^\pi \frac{\sin x}{1+\cos^2 x} dx$$

Put $\cos x = t$
$\sin x dx = -dt$

$$\begin{aligned} I &= -\frac{\pi}{2} \int_{+1}^{-1} \frac{dt}{1+t^2} \\ &= \frac{\pi}{2} \int_{-1}^{1} \frac{dt}{1+t^2} = \frac{\pi}{2} \cdot 2 \int_0^1 \frac{dt}{1+t^2} \\ &= \pi \left(\tan^{-1} t\right)_0^1 \\ &= \pi \tan^{-1} 1 = \pi \times \frac{\pi}{4} = \frac{\pi^2}{4} \end{aligned}$$

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# Question30

**If $\int_0^{2\pi} \left(\sin^4 x + \cos^4 x\right) dx = K \int_0^\pi \sin^2 x dx + L \int_0^{\frac{\pi}{2}} \cos^2 x dx$ and $K, L \in N$, then the number of possible ordered pairs ( $K, L$ ) is**

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**Options:**

A. 1

B. 2

C. 3

D. 4

**Answer: B**

## Solution:

Let $I = \int_0^{2\pi} \left(\sin^4 x + \cos^4 x\right) dx$

As $\sin^4 x + \cos^4 x$ is periodic with $\frac{\pi}{2}$

So, $\int_0^{2\pi}\left(\sin^4 x+\cos^4 x\right) dx$

$$=\int_0^{4\times\frac{\pi}{2}}\left(\sin^4 x+\cos^4 x\right) dx$$

$$=4 \int_0^{\pi / 2}\left(\sin^4 a x+\cos^4 x\right) d x$$

$$=4 \int_0^{\pi / 2} 1-\frac{\sin^2 2 x}{2} d x$$

$$=4 \int_0^{\pi / 2} \frac{4-1+\cos 4 x}{4} d x$$

$$=4 \int_0^{\pi / 2} \frac{3+\cos 4 x}{4} d x$$

$$=\left[3-\frac{\sin 4 x}{4}\right]_0^{\frac{\pi}{2}}$$

$$=3\left[\frac{\pi}{2}\right]=\frac{3 \pi}{2}$$

Now, $\int_0^\pi \sin^2 x d x=2 \int_0^{\frac{\pi}{2}} \sin^2 x d x$

$$=2 \int_0^{\frac{\pi}{2}}\left(\frac{1-\cos 2 x}{2}\right) d x=\left[x-\frac{\sin 2 x}{2}\right]_0^{\frac{\pi}{2}}$$

$$=\frac{\pi}{2}$$

Similarly,

$$\int_0^{\frac{\pi}{2}} \cos^2 x d x=\frac{\pi}{4}$$

$$\therefore \frac{3 \pi}{2}=K \times \frac{\pi}{2}+L \times \frac{\pi}{4}$$

$$2 K+L=6$$

If $K=1$, then $L=4$ and if $K=2$, then $L=2$

$\therefore$ There are two possible ordered pair i.e. $(1,4),(2,2)$.

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# Question31

**$\int_0^\pi \frac{x \sin x}{4 \cos^2 x+3 \sin^2 x} d x$ is equal to**

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**Options:**

A. $\frac{\pi^2}{6 \sqrt{3}}$

B. $\frac{\pi}{3\sqrt{3}}$

C. $\frac{\pi^2}{3\sqrt{3}}$

D. $\sqrt{3}\pi^2$

**Answer: A**

## Solution:

$$\begin{aligned} Let\ I &= \int_0^\pi \frac{x \sin x dx}{4\cos^2 x + 3\sin^2 x} \quad \ldots(i) \\ &= \int_0^\pi \frac{x \sin x dx}{4\cos^2 x + 3 - 3\cos^2 x} \\ &= \int_0^\pi \frac{x \sin x}{\cos^2 x + 3} \\ &= \int_0^\pi \frac{(\pi - x)\sin(\pi - x)}{\cos^2(\pi - x) + 3} \end{aligned}$$

$$= \int_0^\pi \frac{(\pi - x)\sin x}{\cos^2 x + 3} \quad \ldots(ii)$$

On adding Eq. (i) and (ii), we get

$$\begin{aligned} 2I &= \int_0^\pi \frac{\pi \sin x}{3 + \cos^2 x} dx \\ &= \pi \int_0^\pi \frac{\sin x}{3 + \cos^2 x} dx = -\pi \int_1^{-1} \frac{1}{3 + t^2} dt \\ &= -\pi \frac{1}{\sqrt{3}} \left(\tan^{-1} \frac{t}{\sqrt{3}}\right)_1^{-1} \\ \Rightarrow \quad 2I &= \frac{-\pi}{\sqrt{3}} \left[\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right] \\ &= \frac{-\pi}{\sqrt{3}} \left[-\frac{\pi}{6} - \frac{\pi}{6}\right] \\ &= -\frac{\pi}{\sqrt{3}} \times \left(-\frac{2\pi}{6}\right) = \frac{\pi^2}{3\sqrt{3}} \\ \Rightarrow \quad I &= \frac{\pi^2}{6\sqrt{3}} \end{aligned}$$

---

# Question32

**If $A = \int_0^\infty \frac{1+x^2}{1+x^4} dx, B = \int_0^1 \frac{1+x^2}{1+x^4} dx$, then**

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**Options:**

A. $2A = B$

B. $A = B$

C. $2B = A$

D. $2B + A = 0$

**Answer: C**

## Solution:

$$\text{Let } I = \int \left(\frac{1+x^2}{1+x^4}\right) dx$$
$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}-2+2\right)} dx$$
$$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2} dx$$

Let $x - \frac{1}{x} = t$

$$\Rightarrow \left(1+\frac{1}{x^2}\right) dx = dt$$
$$= \int \frac{1}{t^2+2} dt = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right)$$
$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) \text{ or } \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)$$

$$A = \int_0^\infty \frac{1+x^2}{1+x^4} dx = \left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)\right]_0^\infty$$
$$= \frac{1}{\sqrt{2}}\left[\tan^{-1}\infty - \tan^{-1}(-\infty)\right]$$
$$= \frac{1}{\sqrt{2}} \times 2 \times \frac{\pi}{2} \Rightarrow \frac{\pi}{\sqrt{2}}$$
$$B = \int_0^1 \frac{1+x^2}{1+x^4} dx = \left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)\right]_0^1$$
$$= \frac{1}{\sqrt{2}}\left[\tan^{-1}(0) - \tan^{-1}(-\infty)\right] = \frac{1}{\sqrt{2}} \times \frac{\pi}{2}$$
$$\Rightarrow A = 2B$$

---

# Question33

$\int_0^1 \sqrt{\frac{2+x}{2-x}} dx$ **is equal to**

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**Options:**

A. $\pi+2$

B. $\frac{1}{2}(\pi+2)$

C. $\frac{\pi}{2}+2+\sqrt{3}$

D. $\frac{\pi}{3}+2-\sqrt{3}$

**Answer: D**

## Solution:

We have, $I=\int_0^1 \sqrt{\frac{2+x}{2-x}} dx$

Let $\frac{2+x}{2-x}=u$

$\left(\frac{1}{2-x}+\frac{x+2}{(2-x)^2}\right) dx=du$

Now, lower limit $=1$

and upper limit $=3$

$=\int_1^3 \frac{\sqrt{u}}{(u+1)^2} du$

Now, let $s=\sqrt{u}$

$$
\begin{aligned}
ds=\frac{1}{2 \sqrt{u}} \cdot du & =8 \int_1^{\sqrt{3}} \frac{s^2}{\left(s^2+1\right)^2} ds \\
& =8 \int_1^{\sqrt{3}}\left(\frac{1}{s^2+1}-\frac{1}{\left(s^2+1\right)^2}\right) ds \\
& =8\left|\tan ^{-1}(s)\right|_1^{\sqrt{3}}-8 \int_1^{\sqrt{3}} \frac{1}{\left(1+s^2\right)^2} ds \\
& =8\left(\tan ^{-1}(\sqrt{3})-\tan ^{-1}(1)\right)-8
\end{aligned}
$$

$\int_1^{\sqrt{3}} \frac{1}{\left(1+s^2\right)^2} ds$

$$= 8\left(\frac{\pi}{3} - \frac{\pi}{4}\right) - 8\int_1^{\sqrt{3}} \frac{1}{(1+s^2)^2} ds$$
$$= \frac{2\pi}{3} - 8\int_{\pi/4}^{\pi/3} \cos^2 p dp$$

$$[\text{put } s = \tan p]$$

$$= \frac{2\pi}{3} + 2 - \sqrt{3} + 1 - 4p|_{\pi/4}^{\pi/3}$$
$$= \frac{1}{3}(6 - 3\sqrt{3} + \pi) = \frac{\pi}{3} + 2 - \sqrt{3}$$

---

# Question34

**If $M = \int\limits_0^\infty \frac{\log t}{1+t^3} dt$ and $N = \int\limits_{-\infty}^\infty \frac{te^{2t}}{1+e^{3t}} dt$, then**

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**Options:**

A. $N = 2M$

B. $N = M$

C. $N = 3M$

D. $N = -M$

**Answer: D**

## Solution:

We have,

$$M = \int_0^\infty \frac{\log t}{1+t^3} \text{ and } N = \int_{-\infty}^\infty \frac{te^{2t}}{1+e^{3t}} dt$$
$$N = \int_{-\infty}^\infty \frac{t \cdot e^{2t}}{1+e^{3t}} dt$$

Let $e^t = x$

$e^t \cdot dt = dx$

Lower limit = 0

and Upper limit $= \infty$

$$N=\int_0^{\infty}\frac{\log x\cdot x^2}{1+x^3}\cdot\frac{dx}{x}=\int_0^{\infty}\frac{x\log x}{1+x^3}dx$$

$$=\int_0^{\infty}\frac{x\log x}{1+x^3}dx=-\int_0^{\infty}\frac{\log x}{1+x^3}dx$$

$$\Rightarrow N=-\int_0^{\infty}\frac{\log t}{1+t^3}dt\Rightarrow N=-M$$

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# Question35

$\int\limits_{-2}^{2}\left(4-x^2\right)^{\frac{5}{2}}dx$ **is equal to**

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**Options:**

A. $40\pi$

B. $20\pi$

C. $10\pi$

D. $\frac{5\pi}{32}$

**Answer: B**

## Solution:

To solve the integral $I=\int_{-2}^{2}\left(4-x^2\right)^{\frac{5}{2}}dx$, we can use a substitution method.

**Substitution:**

**Substitute** $x=2\sin u$:

Then $dx=2\cos u\,du$.

**Change the limits:**

When $x=-2$, $u=-\frac{\pi}{2}$.

When $x=2$, $u=\frac{\pi}{2}$.

**Substitute into the integral:**

$I=\int_{-\pi/2}^{\pi/2}\left(4-(2\sin u)^2\right)^{\frac{5}{2}}\cdot 2\cos u\,du$

$= \int_{-\pi/2}^{\pi/2} \left(4 - 4\sin^2 u\right)^{\frac{5}{2}} \cdot 2\cos u \, du$

**Simplify:**

Since $4 - 4\sin^2 u = 4\cos^2 u$, we have:

$= \int_{-\pi/2}^{\pi/2} \left(4\cos^2 u\right)^{\frac{5}{2}} \cdot 2\cos u \, du$

$= 32 \int_{-\pi/2}^{\pi/2} \cos^6 u \, du$

**Using Reduction Formula:**

Solving this integral using the reduction formula yields:

$I = 64 \int_{-\pi/2}^{\pi/2} \cos^6 u \, du = 20\pi$

Thus, the value of the integral is $I = 20\pi$.

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# Question36

$\lim_{x \to \infty} \left[ \left(1 + \frac{1}{n^3}\right)^{\frac{1}{n^3}} \left(1 + \frac{8}{n^3}\right)^{\frac{4}{n^3}} \left(1 + \frac{27}{n^3}\right)^{\frac{9}{n^3}} \ldots\ldots (2)^{\frac{1}{n}} \right]$ is equaln

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**Options:**

A. $\log 2 - \frac{1}{2}$

B. $e^{\left(\log 2 - \frac{1}{2}\right)}$

C. $e^{\left(\frac{2\log 2 - 1}{3}\right)}$

D. $\frac{1}{3}(2\log 2 - 1)$

**Answer: C**

## Solution:

We have, $P = \lim_{x \to \infty}$

$$\left[\left(1+\frac{1}{n^3}\right)^{1/n^3} - \left(1+\frac{8}{n^3}\right)^{4/n^3} \ldots (2)^{1/n}\right]$$

$$\Rightarrow \quad \log p = \lim_{x \to \infty} \frac{1}{n} \sum_{r=1}^{n} \frac{r^2}{n^2} \log\left(1+\frac{r^3}{n^3}\right)$$

$$\Rightarrow \quad \log p = \int_0^1 x^2 \log\left(1+x^3\right) dx$$

Put $1+x^3 = t \Rightarrow 3x^2 dx = dt$

when $n = 0, t = 1$ and $x = 1, t = 2$

$$\therefore \log p = \frac{1}{3} \int_1^2 \log t dt = \frac{1}{3}[t \log t - t]_1^2$$

$$\Rightarrow \log p = \frac{1}{3}[2 \log 2 - 1]$$

Hence, $p = e^{(2 \log 2 - 1/3)}$

---

# Question37

$$\int_{-5\pi}^{5\pi} (1 - \cos 2x)^{\frac{5}{2}} dx \text{ is equal to}$$

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**Options:**

A. $\frac{64\sqrt{2}}{5}$

B. $\frac{128\sqrt{2}}{5}$

C. $\frac{256\sqrt{2}}{3}$

D. $\frac{128\sqrt{2}}{3}$

**Answer: D**

## Solution:

To solve the integral $\int_{-5\pi}^{5\pi}(1-\cos 2x)^{\frac{5}{2}} dx$, we start by simplifying the expression inside the integral:

Notice that $1 - \cos 2x = 2\sin^2 x$. Substituting this in, the integral becomes:

$I = \int_{-5\pi}^{5\pi}(1-\cos 2x)^{\frac{5}{2}} dx = \int_{-5\pi}^{5\pi}(2\sin^2 x)^{\frac{5}{2}} dx$

Simplify the expression:

$= \int_{-5\pi}^{5\pi} 2^{\frac{5}{2}} \cdot |\sin^5 x| \, dx$

Since $|\sin^5 x| = \sin^5 x$ for the interval $[0, \pi]$ and is symmetrical over $[-\pi, 0]$, the integral becomes:

$= 8\sqrt{2} \int_0^{5\pi} \sin^5 x \, dx$

Exploit the periodicity of the sine function to break the integral over one full period:

$= 40\sqrt{2} \int_0^{\pi} \sin^5 x \, dx$

Using the reduction formula for $\int_0^{\pi} \sin^n x \, dx$, the final result of this reduction gives:

$= \frac{128\sqrt{2}}{3}$

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# Question38

$\int_0^{\pi/4} \log(1 + \tan x) dx =$

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**Options:**

A. $\pi \log 2 + 1$

B. $\frac{\pi}{2} \log 2 + 1$

C. $\frac{\pi}{4} \log 2$

D. $\frac{\pi}{8} \log 2$

**Answer: D**

## Solution:

$I = \int_0^{\pi/4} \log(1 + \tan x) dx \qquad \ldots (i)$

Using $\int_0^a f(x) dx = \int_0^a f(a - x) dx$

$$I=\int_0^{\pi/4}\log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$$

$$=\int_0^{\pi/4}\log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$$

$$\left(\because \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}\right)$$

$$I=\int_0^{\pi/4}\log\left(\frac{2}{1+\tan x}\right)dx \qquad \ldots(ii)$$

On adding Eq. (i) and Eq. (ii), we get

$$2I=\int_0^{\pi/4}\log(1+\tan x)dx$$

$$+\int_0^{\pi/4}\log\frac{2}{(1+\tan x)}dx$$

$$=\int_0^{\pi/4}\log(1+\tan x)\cdot\frac{2}{(1+\tan x)}dx$$

$$=\int_0^{\pi/4}\log 2dx=\log 2\int_0^{\pi/4}dx$$

$$=\log 2\cdot(x)_0^{\pi/4}=\log 2\cdot\left(\frac{\pi}{4}-0\right)$$

$$2I=\frac{\pi}{4}\log 2\Rightarrow I=\frac{\pi}{8}\log 2$$

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# Question39

$$\int_{\pi}^{\pi}\frac{x\sin x}{1+\cos^2 x}dx=$$

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**Options:**

A. $\frac{3\pi^2}{4}$

B. $\frac{\pi}{2}+1$

C. $\frac{\pi^2}{4}$

D. $\frac{\pi^2}{2}$

**Answer: D**

## Solution:

$$I=\int_{-\pi}^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$
$$=2\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$
$$=2\left[\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx\right]$$
$$=2\left[\pi\int_0^{\pi}\frac{\sin xdx}{1+\cos^2 x}-\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}\right]$$
$$2I=2\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

$$Let\quad t=\cos x$$
$$dt=-\sin xdx$$
$$=-\pi\int_1^{-1}\frac{1}{1+t^2}dt=\pi\int_{-1}^{1}\frac{1}{1+t^2}dt$$
$$I=\pi\left[\tan^{-1}(t)\right]_{-1}^{1}$$
$$=\pi\left[\frac{\pi}{4}+\frac{\pi}{4}\right]=\frac{\pi^2}{2}$$

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# Question40

$$\int_0^{\pi/4}\frac{x^2}{(x\ \sin\ x+\cos\ x)^2}dx=$$

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**Options:**

A. $\frac{2-\pi}{2+\pi}$

B. $\frac{4-\pi}{4+\pi}$

C. $\frac{6-\pi}{6+\pi}$

D. $\frac{8-\pi}{8+\pi}$

**Answer: B**

## Solution:

We have,

$$I=\int_0^{\pi/4}\frac{x^2dx}{(x\sin x+\cos x)}\Rightarrow I=\int_0^{\pi/4}\frac{x\sec x\cdot\cos x\cdot dx}{(x\sin x+\cos x)^2}$$

$$\Rightarrow\quad I=\int_0^{\pi/4}x\sec x\cdot\frac{x\cos x}{(x\sin x+\cos x)^2}dx$$

Let $I_1=x\sec x$ and $I_2=\frac{x\cos x}{x\sin x+\cos x}$

Let $x\sin x+\cos x=t$

$$x\cos x+\sin x-\sin x=\frac{dt}{dx}$$
$$x\cos xdx=dt$$

$$\Rightarrow\qquad I_2=\int\frac{x\cos x}{(x\sin x+\cos x)^2}=\int\frac{dt}{t^2}=-\frac{1}{t}$$

$$\Rightarrow\qquad I_2=\frac{-1}{(x\sin x+\cos x)}$$

$$\Rightarrow\qquad I=\int_0^{\pi/4}x\sec x\cdot\frac{x\cos x}{(x\sin x+\cos x)}dx$$

$$\Rightarrow\qquad I=\left[x\sec x\cdot\frac{-1}{(x\sin x+\cos x)}\right]_0^{\frac{\pi}{4}}-\int_0^{\pi/4}[\sec x+x\sec x\tan x]\cdot\frac{-1}{(x\sin x+\cos x)}dx$$

$$\because\qquad\int f(x)g(x)dx=f(x)\int g(x)dx-\int\left[\frac{d}{dx}f(x)-\int g(x)dx\right]dx$$

$$\Rightarrow\quad I=\left[\frac{-x\sec x}{x\sin x+\cos x}\right]_0^{\pi/4}+\int_0^{\pi/4}\frac{\sec x(1+x\tan x)dx}{(x\sin x+\cos x)}$$

$$\Rightarrow\quad I=\left[\frac{-x\sec x}{x\sin x+\cos x}\right]_0^{\pi/4}+\int_0^{\pi/4}\frac{\sec^2 x(\cos x+x\sin x)}{(x\sin x+\cos x)}dx$$

$$\Rightarrow\quad I=\left[\frac{-x\sec x}{(x\sin x+\cos x)}\right]_0^{\pi/4}+\int_0^{\pi/4}\sec^2 xdx$$

$$\Rightarrow\quad I=\frac{-\frac{\pi}{4}\cdot\sqrt{2}}{\frac{\pi}{4}\cdot\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}+\int_0^{\pi/4}\sec^2 xdx$$

$$\Rightarrow\quad I=\frac{-\pi\sqrt{2}/4}{(\pi+4)/4\sqrt{2}}+[\tan x]_0^{\pi/4}$$

$$\Rightarrow\quad I=\frac{-2\pi}{\pi+4}+(1-0)=\frac{-2\pi}{\pi+4}+1\Rightarrow I=\frac{4-\pi}{\pi+4}$$

---

# Question41

$$\int_0^1\frac{x}{(1-x)^{\frac{3}{4}}}dx=$$

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**Options:**

A. $\frac{4}{5}$

B. $\frac{8}{15}$

C. $\frac{14}{5}$

D. $\frac{16}{5}$

**Answer: D**

## Solution:

We have, $I=\int_0^1 \frac{x}{(1-x)^{\frac{3}{4}}} dx$

Let $1-x=t \Rightarrow x=1-t \Rightarrow -dx=dt$

$$\therefore \quad I=\int_0^1 \frac{1-t}{t^{3/4}} dt \Rightarrow I=\int_0^1 t^{-3/4}(1-t) dt$$

$$\Rightarrow \quad I=\int_0^1 t^{-3/4} dt-\int_0^1 t^{1/4} dt \Rightarrow I=\left[4 t^{1/4}\right]_0^1-\left[\frac{4 t^{5/4}}{5}\right]_0^1$$

Putting the value of $t$

$I=[4(1-x)]_0^1+\left[\frac{4}{5}(1-x)^{5/4}\right]_0^1$

On solving, we get

$I=\frac{16}{5}$

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# Question42

$$\int_{-1}^1\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right) dx=$$

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**Options:**

A. 2

B. 4

C. 0

D. 8

**Answer: C**

## Solution:

We have,

$I=\int_{-1}^{1}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)dx$

Let $f(x)=\sqrt{1+x+x^2}-\sqrt{1-x+x^2}$

$\Rightarrow f(-x)=\sqrt{1-x+x^2}-\sqrt{1+x+x^2}$
$\Rightarrow f(-x)=-\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)\Rightarrow f(-x)=-f(x)$

$\because f(x)$ is an odd function.

As, we know that

$\int_{-a}^{a}f(x)dx=\left\{\begin{array}{l}\int_0^{2a}f(x)dx,\text{ if } f(-x)=f(x),\text{ even function}\\ 0,\text{ if } f(-x)=-f(x),\text{ odd funciton}\end{array}\right\}$

Then, $I=\int_{-1}^{1}\sqrt{1+x+x^2}$
$-\sqrt{1-x+x^2}dx=0$

---

# Question43

$\int_1^5(|x-3|+|1-x|)dx=$

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**Options:**

A. 4

B. 8

C. 12

D. 24

**Answer: C**

## Solution:

We have,

$\Rightarrow\quad I=\int_1^5|x-3|+|1-x|dx$

Let $f(x)=|x-3|+|1-x|$

Here, when $1 \leq x \leq 3$

$\Rightarrow f(x)=-x-3-1+x=2$

When, $3 \leq x \leq 5$

$\Rightarrow f(x)=x-3-1+x=2x-4$

$$\begin{aligned} \Rightarrow \quad I &= \int_1^5 |x-3|+|1-x| dx \\ &= \int_1^3 2dx+\int_3^5 (2x-4) dx \\ \Rightarrow \quad I &= [2x]_1^3+\left[\frac{2x^2}{2}-4x\right]_3^5 \end{aligned}$$

On solving, we get

$I=12$

---

# Question44

**If 729 $\int_1^3 \frac{1}{x^3\left(x^2+9\right)^2} dx=a+\log b$, then $(a-b)=$**

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**Options:**

A. 4

B. $-\frac{4}{5}$

C. $\frac{4}{5}$

D. -4

**Answer: A**

## Solution:

If 729 $\int_1^3 \frac{1}{x^3\left(x^2+9\right)^2} dx=a+\log b$ Then, $a-b$

On substituting $u=x^2$

$\Rightarrow \qquad du = 2x dx$

$\Rightarrow \qquad dx = \frac{1}{2}\frac{du}{\sqrt{u}}$

when, $x = 1$

$\Rightarrow \quad u = 1$

$x = 3$

$\Rightarrow \quad u = 9$

$$I = 729 \int_1^9 \frac{1}{u^{\frac{3}{2}}(u+9)^2} du$$

$$= 729 \int_1^9 \frac{1}{2u^{\frac{3}{2}+\frac{1}{2}}(u+9)^2} du$$

$$= 729 \int_1^9 \frac{1}{2u^2(u+9)^2} du$$

Partial fraction

$$\frac{1}{u^2(u+9)^2} = \frac{A}{u} + \frac{B}{u^2} + \frac{C}{(u+9)} + \frac{D}{(u+9)}$$

$$\frac{1}{u^2(u+9)^2} \frac{A\left(u(u+9)^2 + B(u+9)^2 + Cu^2(u+9) + Du^2\right.}{u^2(u+9)^2}$$

$$1 = Au\left(u^2 + 81 + 18u\right) + B\left(u^2 + 81 + 18u\right) + Cu^3 + 9Cu^2 + Du^2$$

$$1 = Au^3 + 81Au + 18Au^2 + Bu^2 + 81B + 18uB + Cu^3 + 9Cu^2 + Du^2$$

$$1 = (A+C)u^3 + (18A + B + 9C + D)u^2 + (81A + 18B)u + 81B$$

Comparison of coefficient $u^3, u^2, u$ and 1.

$A + C = 0, 18A + B + 9C + D = 0$

$81A + 18B = 0 \Rightarrow 81B = 1$

$$B = \frac{1}{81} \Rightarrow 81A + 18 \cdot \frac{1}{81} = 0$$

$$81A = -\frac{2}{9} \Rightarrow A = \frac{-2}{729}$$

$$\frac{-2}{729} + C = 0 \Rightarrow C = \frac{2}{729}$$

$$18\left(\frac{-2}{729}\right) + \frac{1}{81} + 9\left(\frac{2}{729}\right) + D = 0$$

$$-\frac{4}{81} + \frac{1}{81} + \frac{2}{81} + D = 0 \Rightarrow \frac{-1}{81} + D = 0$$

$$D = \frac{1}{81}$$
$$= \frac{729}{2} \int_1^9 \left( \left( \frac{-2}{729} \right) \frac{1}{u} + \frac{1}{81u^2} + \frac{2}{729(u+9)} + \frac{1}{81(u+9)^2} \right) du$$
$$= \frac{729}{2} \left[ \frac{-2}{729} \log u + \frac{1}{81} \frac{u^{-2+1}}{(-2+1)} + \frac{2}{729} \log(u+9) + \frac{1}{81} \frac{(u+9)^{-2+1}}{-2+1} \right]_1^9$$
$$= \frac{729}{2} \left[ \frac{-2}{729} (\log 9 - \log 1) + \frac{1}{81} (-1) \left( \frac{1}{9} - 1 \right) + \frac{2}{729} [\log(18) - \log(10)] \right.$$
$$= \frac{729}{2} \left[ \frac{-2}{729} \log 9 - \frac{1}{81} \left( \frac{-8}{9} \right) + \frac{2}{729} \log \left( \frac{18}{10} \right) - \frac{1}{81} \left( \frac{-8}{180} \right) \right] + C$$
$$= \frac{1}{2} \left[ -2 \log 9 + 8 + 2 \log \left( \frac{1}{5} \right) + \frac{8}{20} \right] + C$$
$$= \frac{1}{2} \left[ -2 \log 9 + 8 + 2 \log 9 - 2 \log 5 + \frac{2}{5} \right] + C$$
$$= \frac{1}{2} \left[ 8 + \frac{2}{5} - 2 \log 5 \right] + C$$
$$= \frac{1}{2} \left[ \frac{40+2}{5} - 2 \log 5 \right] \Rightarrow \frac{42}{2 \times 5} - \log 5$$

$$a = \frac{42}{2 \times 5}, b = \frac{1}{5}$$
$$a = \frac{21}{5}, b = \frac{1}{5}$$
$$a - b = \frac{21}{5} - \frac{1}{5} = \frac{20}{5} = 4$$

---

# Question45

$$\lim_{n \to \infty} \frac{1^{17} + 2^{77} + \ldots + n^{77}}{n^{78}} =$$

## AP EAPCET 2024 - 21th May Morning Shift

**Options:**

A. $\frac{1}{77}$

B. 1

C. 76

D. $\frac{1}{78}$

**Answer: D**

## Solution:

To find the limit

$\lim_{n \to \infty} \frac{1^{77}+2^{77}+...+n^{77}}{n^{78}}$

we use the method of integral approximation to approximate the sum of powers. Consider the sum

$S_n = 1^{77} + 2^{77} + \ldots + n^{77}.$

This can be approximated using the integral

$S_n \approx \int_1^n x^{77}\, dx.$

Evaluating this integral, we have

$\int_1^n x^{77}\, dx = \left[\frac{x^{78}}{78}\right]_1^n = \frac{n^{78}}{78} - \frac{1}{78}.$

As $n$ becomes very large, the term $\frac{1}{78}$ becomes negligible, and we approximate

$S_n \approx \frac{n^{78}}{78}.$

Thus, the given limit can be rewritten and evaluated as follows:

$\lim_{n \to \infty} \frac{S_n}{n^{78}} = \lim_{n \to \infty} \frac{\frac{n^{78}}{78}}{n^{78}} = \lim_{n \to \infty} \frac{1}{78} = \frac{1}{78}.$

Therefore, the value of the limit is

$\frac{1}{78}.$

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# Question46

If $f(x) = \begin{cases} \frac{6x^2+1}{4x^3+2x+3} & ,0 < x < 1 \\ x^2+1 & ,1 \leq x < 2 \end{cases}$, then $\int_0^2 f(x)dx =$

## AP EAPCET 2024 - 21th May Morning Shift

**Options:**

A. $\frac{1}{2}\log 3 + \frac{10}{3}$

B. $\frac{1}{2}\log 3 - \frac{10}{3}$

C. $\frac{1}{2}\log 3 + \frac{13}{3}$

D. $\frac{1}{2}\log 3 + \frac{20}{3}$

**Answer: A**

## Solution:

$$f(x)=\begin{cases}\frac{6x^2+1}{4x^3+2x+3}, & 0<x<1\\ x^2+1 & 1\le x<2\end{cases}$$

$$\int_0^2 f(x)dx=\int_0^1\left(\frac{6x^2+1}{4x^3+2x+3}\right)dx+\int_1^2\left(x^2+1\right)dx$$

Let $u=4x^3+2x+3$

$$du=\left(12x^2+2\right)dx\Rightarrow 2\left(6x^2+1\right)dx$$

$$\Rightarrow dx=\frac{du}{2\left(6x^2+1\right)}$$

$$x=0\Rightarrow u=3$$

$$x=1\Rightarrow u=4+2+3=9$$

Or $\left(6x^2+1\right)dx=\frac{du}{2}\Rightarrow\int_3^9\frac{1}{u}\left(\frac{du}{2}\right)+\int_1^2\left(x^2+1\right)dx$

$$\frac{1}{2}[\log u]_3^9+\left[\frac{x^3}{3}+x\right]_1^2$$

$$\frac{1}{2}[\log 9-\log 3]+\left[\frac{8}{3}+2-\frac{1}{3}-1\right]$$

$$\frac{1}{2}\left[\log\left(\frac{9}{3}\right)+\left(\frac{8+6}{3}-\frac{4}{3}\right)\right]$$

$$\frac{1}{2}\log 3+\frac{14}{3}-\frac{4}{3}=\frac{1}{2}\log 3+\frac{10}{3}$$

---

# Question47

### If $\int_1^n [x]dx=120$, then $n=$

### AP EAPCET 2024 - 21th May Morning Shift

**Options:**

A. 15

B. 16

C. 14

D. 12

**Answer: B**

## Solution:

We have,

$$\int_1^n [x]dx = 120$$

$$= \int_1^2 1^{dx} + \int_2^3 2^{dx} + \ldots + \int_{n-1}^n (n-1)dx = 120$$

$$\Rightarrow \quad [x]_1^2 + [2x]_2^3 + \ldots [(n-1)x]_{n-1}^n = 120$$

$$\Rightarrow \quad 2 - 1 + 6 - 4 + \ldots + (n-1) = 120$$

$$\Rightarrow \quad 1 + 2 + \ldots + n - 1 = 120$$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 120$$

$$\Rightarrow \quad n^2 - n - 240 = 0$$

$$\Rightarrow \quad n^2 - 16n + 15n - 240 = 0$$

$$\Rightarrow \quad n(n-16) + 15(n-16) = 0$$

$$(n-16)(n+15) = 0$$

$$n = 16$$

---

# Question48

$$\int_{\frac{-1}{24}}^{\frac{1}{24}} \sec x \log\left(\frac{1-x}{1+x}\right) dx =$$

## AP EAPCET 2024 - 20th May Evening Shift

**Options:**

A. $\frac{\pi}{2}$

B. $\pi$

C. 1

D. 0

**Answer: D**

## Solution:

$$\int_{-1/24}^{1/24} \sec x \log\left(\frac{1-x}{1+x}\right) dx$$

Here, $f(x)=\sec x \log \left(\frac{1-x}{1+x}\right)$

Now, $f(-x)=\sec (-x) \log \left(\frac{1-(-x)}{1-x}\right)$

$$=\sec x \log \left(\frac{1+x}{1-x}\right)$$
$$=-\sec x \log \left(\frac{1-x}{1+x}\right)=-f(x)$$

Thus, the given function is an odd function

$$\therefore \quad \int\limits_{\frac{-1}{24}}^{\frac{1}{24}} \sec x \log \left(\frac{1-x}{1+x}\right) d x=0$$

$\left[\because \int\limits_{-a}^{a} f(x) d x=0\right.$, if $f(x)$ is an odd function]

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# Question49

## If $[x]$ is the greatest integer function, then $\int_0^5[x] d x=$

## AP EAPCET 2024 - 20th May Evening Shift

**Options:**

A. 15

B. 2

C. 3

D. 10

**Answer: D**

## Solution:

$I=\int_0^5[x] d x$

$=\int_0^1 0 d x+\int_1^2 1 d x+\int_2^3 2 d x+\int_3^4 3 d x+\int_4^5 4 d x$

$=0+(2-1)+2(3-2)+3(4-3)+4(5-4)=1+2+3+4=10$

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# Question50

$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}} dx =$

### AP EAPCET 2024 - 20th May Evening Shift

**Options:**

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

**Answer: D**

## Solution:

$I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}} dx \quad \ldots \text{(i)}$

$$= \int_0^{\pi/2} \frac{1}{\sqrt{1+\tan\left(\frac{\pi}{2}-x\right)}} dx$$

$$= \int_0^{\pi/2} \frac{1}{\sqrt{1+\cot x}} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} dx \quad \ldots \text{(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$2I = \int_0^{\pi/2} 1 dx = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

---

# Question51

$\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx =$

### AP EAPCET 2024 - 20th May Evening Shift

**Options:**

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi^2}{2}$

D. $\frac{\pi^2}{4}$

**Answer: D**

## Solution:

To solve the integral $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$, we can use symmetry properties of definite integrals. According to the property:

$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

Apply this property to transform the integral:

$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}\, dx$

Adding both expressions for $I$, we have:

$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx$

Now, perform substitution to evaluate the new integral. Let $\cos x = t$, then $-\sin x\, dx = dt$. The limits of integration change as $x$ changes from 0 to $\pi$, causing $t$ to vary from 1 to -1. Thus, we have:

$-\pi \int_1^{-1} \frac{dt}{1+t^2}$

Reversing the limits, the expression becomes:

$\pi \int_{-1}^{1} \frac{dt}{1+t^2}$

This integral evaluates to:

$\pi \left[\tan^{-1} t\right]_{-1}^{1}$

Simplifying:

$= \pi \left(\tan^{-1}(1) - \tan^{-1}(-1)\right)$

Since $\tan^{-1}(1) = \frac{\pi}{4}$, it follows that:

$= \pi \left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \pi \times \frac{\pi}{2} = \frac{\pi^2}{2}$

Thus, the original integral $I$ is:

$I = \frac{1}{2} \times \frac{\pi^2}{2} = \frac{\pi^2}{4}$

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# Question52

$\int_{-\pi}^{\pi} \frac{x \sin^3 x}{4-\cos^2 x} dx =$

## AP EAPCET 2024 - 20th May Morning Shift

**Options:**

A. $2\pi(1 - \log 3)$

B. $2\pi\left(1 - \frac{3}{4}\log 3\right)$

C. $\pi\left(1 - \frac{3}{4}\log 3\right)$

D. $4\pi(1 - \log 3)$

**Answer: B**

## Solution:

We have $I = \int_{-\pi}^{\pi} \frac{x \sin^3 x}{4-\cos^2 x} dx$

Let $f(x) = \frac{x \sin^3 x}{4-\cos^2 x}$

$f(-x) = \frac{-x \times \sin^3(-x)}{4-\cos^2(-x)} = \frac{x \sin^3 x}{4-\cos^2 x} = f(x)$

$\therefore f(x)$ is an even function

$$I = 2\int_0^{\pi} \frac{x \sin^3 x}{4 - \cos^2 x} dx$$
$$I = 2\int_0^{\pi} \frac{(\pi - x)\sin^3 x}{4 - \cos^2 x} dx$$

On adding Eqs. (ii) and (ii), we get

$$2I = 2\pi \int_0^{\pi} \frac{\sin^3 x}{4 - \cos^2 x} dx$$
$$I = \pi \int_0^{\pi} \frac{\sin^3 x}{4 - \cos^2 x} dx$$

Let, $\cos x = t \Rightarrow -\sin x dx = dt$

When $x \rightarrow 0 t \rightarrow 1$ and $x \rightarrow \pi, t \rightarrow -1$

$$
\begin{aligned}
I &= -\pi \int_1^{-1} \frac{\left(1-t^2\right)}{4-t^2} dt \\
&= \pi \int_{-1}^1\left(\frac{-3}{4-t^2}+1\right) dt \\
&= \pi\left[\frac{-3}{4} \log \left|\frac{2+t}{2-t}\right|+t\right]_{-1}^1 \\
&= \pi\left[\left(\frac{-3}{4} \log 3+1\right)-\left(-\frac{3}{4} \log \frac{1}{3}-1\right)\right] \\
&= \pi\left[-\frac{3}{4} \log 3+1-\frac{3}{4} \log 3+1\right] \\
&= 2 \pi\left[1-\frac{3}{4} \log 3\right]
\end{aligned}
$$

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# Question53

$$\int_{-3}^{3} |2-x| dx =$$

## AP EAPCET 2024 - 20th May Morning Shift

**Options:**

A. 12

B. 16

C. 13

D. 25

**Answer: C**

## Solution:

We have $I = \int\limits_{-3}^{3} |2 - x| dx$

$$|2 - x| = \begin{cases} (2 - x) & -3 < x < 2 \\ (x - 2) & 2 \leq x < 3 \end{cases}$$

$$\int\limits_{-3}^{3} |2 - x| dx = \int\limits_{-3}^{2} (2 - x) dx + \int\limits_{2}^{3} (x - 2) dx$$

$$= \left[2x - \frac{x^2}{2}\right]_{-3}^{2} + \left[\frac{x^2}{2} - 2x\right]_{2}^{3}$$

$$= (4 - 2) - \left(-6 - \frac{9}{2}\right) + \left(\frac{9}{2} - 6\right) - (2 - 4)$$

$$= 2 + 6 + \frac{9}{2} + \frac{9}{2} - 6 + 2 = 13$$

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# Question54

$$\int_{\frac{1}{\sqrt[5]{31}}}^{\frac{1}{\sqrt[5]{242}}} \frac{1}{\sqrt[5]{x^{30} + x^{25}}} dx =$$

## AP EAPCET 2024 - 20th May Morning Shift

**Options:**

A. $\frac{65}{4}$

B. $\frac{-75}{4}$

C. $\frac{75}{4}$

D. $\frac{-65}{4}$

**Answer: D**

## Solution:

To solve the integral, we have:

$$I = \int_{\frac{1}{\sqrt[5]{31}}}^{\frac{1}{\sqrt[5]{242}}} \frac{1}{\sqrt[5]{x^{30}+x^{25}}} \, dx$$

First, we can rewrite the integrand as:

$$\int_{\frac{1}{\sqrt[5]{31}}}^{\frac{1}{\sqrt[5]{242}}} \frac{1}{x^6 \sqrt[5]{1+\frac{1}{x^5}}} \, dx$$

To simplify, we perform a substitution. Let:

$$1 + \frac{1}{x^5} = t^5$$

Differentiating both sides gives:

$$-\frac{5}{x^6} \, dx = 5t^4 \, dt$$

Thus, the differential becomes:

$$dx = -x^6 \cdot \frac{1}{5t^4} \, dt$$

We need to adjust the limits of integration. When $x \to \frac{1}{\sqrt[5]{31}}$, then $t \to 2$. When $x \to \frac{1}{\sqrt[5]{242}}$, then $t \to 3$.

The integral now becomes:

$$I = -\int_2^3 \frac{1}{t} \times t^4 \, dt$$

This simplifies to:

$$I = -\int_2^3 t^3 \, dt$$

$$I = -\left[\frac{t^4}{4}\right]_2^3$$

Calculating the definite integral gives:

$$I = -\left(\frac{3^4}{4} - \frac{2^4}{4}\right)$$

$$I = -\left(\frac{81}{4} - \frac{16}{4}\right)$$

$$I = -\left(\frac{65}{4}\right)$$

Thus, the evaluated integral is:

$$I = \frac{-65}{4}$$

---

# Question55

**If** $\lim_{n \to \infty} \left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{4}{n^2}\right)\left(1+\frac{9}{n^2}\right)\ldots\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}} = ae^b$**, then**
$a+b=$

### AP EAPCET 2024 - 19th May Evening Shift

**Options:**

A. $\pi-2$

B. $\pi$

C. $\pi+2$

D. $\frac{\pi}{2}$

**Answer: D**

## Solution:

Let,

$P=\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{4}{n^2}\right)\left(1+\frac{9}{n^2}\right)\right.$

$\left.\ldots\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$

$\log P=\lim_{n\to\infty}\frac{1}{n}\left[\log\left(1+\frac{1}{n^2}\right)\right.$

$\left.+\log\left(1+\frac{2^2}{n^2}\right)+\ldots\log\left(1+\frac{n^2}{n^2}\right)\right]$

$\log P=\lim_{n\to-\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)$

$=\int_0^1\log\left(1+x^2\right)dx$

$=\left[x\log\left(1+x^2\right)\right]_0^1-\int_0^1\frac{2x^2}{1+x^2}dx$

$=\log 2-2\int_0^1 dx+2\int_0^1\frac{x^2+1-1}{1+x^2}dx$

$=\log 2-2\int_0^1 dx+2\int_0^1\frac{dx}{1+x^2}$

$=\log 2-zx]_0^1+2\left[\tan^{-1}a\right]^{0^1}$

$\log P=\log 2-2+\frac{\pi}{2}$

$P=e^{\log 2-2+\frac{\pi}{2}}=e^{\log 2}\cdot e^{-2+\frac{\pi}{2}}$

$P=2e^{-2+\frac{\pi}{2}}$

$\therefore\quad a=2\quad b=-2+\frac{\pi}{2}$

$a+b=2-2+\frac{\pi}{2}=\frac{\pi}{2}$

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# Question56

$\int_0^\pi x\sin^4 x\cos^6 x dx=$

## AP EAPCET 2024 - 19th May Evening Shift

**Options:**

A. $\frac{3\pi^2}{512}$

B. $\frac{3\pi^2}{256}$

C. $\frac{\pi^2}{256}$

D. $\frac{\pi^2}{512}$

**Answer: A**

## Solution:

Let $I=\int_0^\pi x\sin^4 x\cos^6 x dx\ldots(i)$

$I=\int_0^\pi(\pi-x)\sin^4 x\cos^6 x dx\ldots(ii)$

On adding Eqs. (i) and (ii), we get

$$I=\pi\int_0^\pi \sin^4 x\times\cos^6 x dx$$

$$I=\frac{\pi}{2}\times 2\int_0^{\frac{\pi}{2}}\sin^4 x\cos^6 x dx$$

$$I=\pi\int_0^{\frac{\pi}{2}}\sin^4 x\cos^6 x dx$$

$$=\frac{\pi\left[\frac{3}{2}\times\frac{1}{2}\times\sqrt{\pi}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\times\sqrt{\pi}\right]}{2\times 120}=\frac{3\pi^2}{512}$$

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# Question57

### If $I_n=\int_0^{\frac{\pi}{4}}\tan^n x dx$, then $I_{13}+I_{11}=$

## AP EAPCET 2024 - 19th May Evening Shift

**Options:**

A. $\frac{1}{13}$

B. $\frac{1}{12}$

C. $\frac{1}{10}$

D. $\frac{1}{11}$

**Answer: B**

## Solution:

To determine $I_n$, we start with the given integral:

$I_n=\int_0^{\frac{\pi}{4}}\tan^n x\, dx$

Rewrite this as:

$I_n = \int_0^{\frac{\pi}{4}} \tan^{n-2} x \times (\sec^2 x - 1)\, dx$

Breaking it down, we have:

$I_n = \int_0^{\frac{\pi}{4}} \tan^{n-2} x \sec^2 x\, dx - \int_0^{\frac{\pi}{4}} \tan^{n-2} x\, dx$

Therefore, we can express this as:

$I_n + I_{n-2} = \int_0^{\frac{\pi}{4}} \tan^{n-2} x \sec^2 x\, dx$

When integrating, we find:

$\int_0^{\frac{\pi}{4}} \tan^{n-2} x \sec^2 x\, dx = \left[\frac{\tan^{n-1} x}{n-1}\right]_0^{\frac{\pi}{4}} = \frac{1}{n-1}$

Hence, we deduce:

$I_n + I_{n-2} = \frac{1}{n-1}$

By setting $n = 13$, it follows that:

$I_{13} + I_{11} = \frac{1}{12}$

Thus, the sum $I_{13} + I_{11}$ is:

$\frac{1}{12}$

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# Question58

$$\lim_{n \to +\infty} \left[\frac{1}{n^4} + \frac{1}{(n^2+1)^{\frac{3}{2}}} + \frac{1}{(n^2+4)^{\frac{3}{2}}} + \frac{1}{(n^2+9)^{\frac{3}{2}}} + \ldots\ldots + \frac{1}{4\sqrt{2}n^5}\right] =$$

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**Options:**

A. $\frac{3}{4\sqrt{2}}$

B. $\frac{3\sqrt{2}}{4}$

C. $\frac{5}{6\sqrt{2}}$

D. $\frac{5\sqrt{2}}{6}$

**Answer: C**

## Solution:

$$\lim_{n\to\infty} x^4 \begin{bmatrix} \frac{1}{x^5} + \frac{1}{(x^2+1)^{5/2}} + \frac{1}{(x^2+4)^{5/2}} \\ + \frac{1}{(x^2+9)^{5/2}} + \ldots + \frac{1}{4\sqrt{2}x^5} \end{bmatrix}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{x^4}{(x^2+r^2)^{5/2}} \left[\because \lim_{n\to\infty} \frac{x^4}{x^5} = 0\right]$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} \left[\frac{1}{\left[1+\left(\frac{r}{n}\right)^2\right]^{5/2}}\right]$$

$$= \int_0^1 \frac{1}{(1+x^2)^{5/2}} dx$$

Let $x = \tan\theta$

$dx = \sec^2\theta d\theta$

$x \to 0, \theta \to 0$ and

$x \to 1, \theta \to \frac{\pi}{4}$

$$= \int_0^{\pi/4} \frac{1}{(1+\tan^2\theta)^{5/2}} \times \sec^2\theta d\theta$$

$$= \int_0^{\pi/4} \frac{1}{\sec^3\theta} d\theta$$

$$= \int_0^{\pi/4} \cos^3\theta d\theta$$

$$= \frac{1}{4} \int_0^{\pi/4} (\cos 3\theta + 3\cos\theta) d\theta$$

$$= \frac{1}{4} \left[\frac{\sin 3\theta}{3} + 3\sin\theta\right]_0^{\pi/4}$$

$$= \frac{1}{4} \left[\frac{1}{3}\sin\left(\pi - \frac{\pi}{4}\right) + 3\sin\frac{\pi}{4}\right]$$

$$= \frac{1}{4} \left[\frac{1}{3} \times \frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}}\right]$$

$$= \frac{1+9}{4\times 3\sqrt{2}} \Rightarrow \frac{10}{4\times 3\sqrt{2}} = \frac{5}{6\sqrt{2}}$$

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# Question59

$$\int_{\log 4}^{\log 4} \frac{e^{2x}+e^x}{e^{2r}-5e^x+6} dx =$$

## AP EAPCET 2024 - 18th May Morning Shift

**Options:**

A. $\log\left(\frac{64}{9}\right)$

B. $\log\left(\frac{256}{81}\right)$

C. $\log\left(\frac{32}{3}\right)$

D. $\log\left(\frac{128}{27}\right)$

**Answer: D**

## Solution:

$$\begin{aligned}\text{Let} \quad I &= \int_{\log 4}^{\log 5} \frac{e^{2x}+e^x}{e^{2x}-5e^x+6} dx \\ &= \int_{\log 4}^{\log 5} \frac{\left(e^x\right)^2+e^x}{\left(e^x-3\right)\left(e^x-2\right)} dx \\ &= \int_{\log 4}^{\log 5} \frac{e^x\left(e^x+1\right)}{\left(e^x-3\right)\left(e^x-2\right)} dx\end{aligned}$$

Let $e^x = t$

$\Rightarrow \quad e^x dx = dt$

When $x \rightarrow \log 4, t \rightarrow 4$

$x \rightarrow \log 5, t \rightarrow 5$

$$\begin{aligned}I &= \int_4^5 \frac{t+1}{(t-3)(t-2)} dt \\ &= \int_4^5 \frac{4}{t-3} - \frac{3}{t-2}\end{aligned}$$

[ $\because$ by partial fraction]

$$\begin{aligned}&= [4\log|t-3| - 3\log|t-2|]_4^5 \\ &= \left[\log\left|\frac{(t-3)^4}{(t-2)^3}\right]_4^5 \\ &= \log\frac{2^4}{3^3} - \log\frac{1}{2^3} = \log\frac{2^4}{3^3} \times 2^3 \\ &= \log\left(\frac{2^7}{3^3}\right) = \log\left(\frac{128}{27}\right)\end{aligned}$$

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# Question60

$\int_1^2 \frac{x^4-1}{x^6-1} dx =$

## AP EAPCET 2024 - 18th May Morning Shift

**Options:**

A. $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

B. $\frac{121}{6}$

C. $\sqrt{2}-1$

D. $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

**Answer: A**

## Solution:

$$\begin{aligned}
\text{Let } I &= \int_1^2 \frac{x^4-1}{x^6-1} dx \\
&= \int_1^2 \frac{\left(x^2-1\right)\left(x^2+1\right)}{\left(x^2-1\right)\left(x^4+x^2+1\right)} dx \\
&= \int_1^2 \frac{x^2+1}{x^4+x^2+1} dx = \int_1^2 \left(\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}\right) dx \\
&= \int_1^2 \left(\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+3}\right) dx \\
&= \int_1^2 \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3} dx
\end{aligned}$$

Let $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right) dx=dt$

When $x \rightarrow 1$, then $t \rightarrow 0$

When $x \rightarrow 2$, then $t \rightarrow \frac{3}{2}$

$$\begin{aligned}
I &= \int_0^{3/2} \frac{1}{t^2+3} dt \\
&= \frac{1}{\sqrt{3}}\left[\tan^{-1} \frac{1}{\sqrt{3}}\right]_0^{3/2} \\
&= \frac{1}{\sqrt{3}} \tan^{-1} \frac{3}{2\sqrt{3}} \\
&= \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2}
\end{aligned}$$

---------------------------------------------------------------------------------------------

# Question61

**Let $T > 0$ be a fixed number. $f : R \rightarrow R$ is a continuous function such that $f(x + T) = f(x), x \in R$If $I = \int\limits_0^T f(x)dx$, then $\int\limits_0^{5T} f(2x)dx =$**

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**Options:**

A. 10 I

B. $\frac{5}{2}I$

C. 5 I

D. 2 I

**Answer: C**

## Solution:

Given, $I = \int\limits_0^T f(x)dx$

If $f(x + T) = f(x)$

Now, $\int\limits_0^{5T} f(2x)dx$

On putting $2x = y$

$\Rightarrow \quad dx = \frac{1}{2}dy$

$\frac{1}{2}\int\limits_0^{10T} f(y)dy = \frac{10I}{2} = 5I$

---

# Question62

$$\int_1^3 x^n \sqrt{x^2 - 1} dx = 6, \text{ then } n =$$

## AP EAPCET 2022 - 5th July Morning Shift

**Options:**

A. 2

B. 3

C. 4

D. 5

**Answer: B**

## Solution:

$$\text{Let } I = \int_1^3 x \sqrt[n]{x^2 - 1} dx = 6$$

$$= \frac{1}{2} \int_1^3 2x \left(x^2 - 1\right)^{\frac{1}{n}} dx = 6$$

$$\Rightarrow \quad \left[ \frac{\left(n^2 - 1\right)^{\frac{1}{n}+1}}{\frac{1}{n} + 1} \right]_1^3 = 12$$

$$\Rightarrow \quad \frac{(8)^{\frac{1}{n}+1}}{\frac{1}{n} + 1} = 12$$

$$\Rightarrow \quad \frac{(8)^{\frac{1}{n}}}{(n+1)}(n) = \frac{3}{2}$$

Now, on putting $n = 3$, L.H.S = R. H. S

$\therefore n = 3$

---

# Question63

**[ . ] represents greatest integer function, then**
$\int_{-1}^{1}(x[1+\sin \pi x]+1)dx =$

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**Options:**

A. 1

B. 2

C. $\frac{5}{2}$

D. $\frac{3}{2}$

**Answer: C**

## Solution:

$$\int_{-1}^{1}(x[1+\sin \pi x]+1)dx$$

$$I=\int_{-1}^{1}(x[1+\sin \pi x]+1)dx$$

We know that, $-1 \leq \sin \pi x \leq 1$

$$I=\int_{-1}^{0}(x[1+\sin \pi x]+1)dx+\int_{0}^{1}(x(1+\sin \pi x)+1)dx$$

$$\Rightarrow I=\int_{-1}^{0}(x \cdot 0+1)dx+\int_{0}^{1}(x+1)dx$$

$$\Rightarrow \quad I=[x]_{-1}^{0}+\left[\frac{x^2}{2}+x\right]_0^1$$

$$\Rightarrow \quad I=0-(-1)+\left[\frac{1}{2}+1\right]$$

$$\Rightarrow \quad I=2+\frac{1}{2}=\frac{5}{2}$$

-----------------------------------------------------------------------------------------------

# Question64

$$\lim_{n\to\infty}\left[\frac{n}{(n+1)\sqrt{2n+1}}+\frac{n}{(n+2)\sqrt{2(2n+2)}}+\frac{n}{(n+3)\sqrt{3(2n+3)}}+\ldots n \text{ terms}\right]=\int_0^1 f(x)dx$$

**then** $f(x)=$

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**Options:**

A. $\frac{1}{(1+x)\sqrt{x^2+2x}}$

B. $\frac{1}{(1+x)\sqrt{x+2}}$

C. $\frac{1}{(1+x)\sqrt{x^2+x+1}}$

D. $\frac{1}{(1+x)\sqrt{x^2-2x}}$

**Answer: A**

## Solution:

Here, $\lim_{n\to\infty}\left[\frac{n}{(n+1)\sqrt{2n+1}}+\frac{n}{(n+2)\sqrt{2(n+2)}}+\ldots \text{ upto } n \text{ terms}\right]$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{n^2}{(n+k)\sqrt{k(2n+k)}}$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{n^2}{(n+k)\sqrt{k(2n+k)}} \times \frac{n^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{\left(\frac{n+k}{n}\right)\sqrt{\frac{k}{n}\left(\frac{2n+k}{n}\right)}}$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{\left(1+\frac{k}{n}\right)\sqrt{\frac{k}{n}\left(2+\frac{k}{n}\right)}}$$

$$= \int_0^1 \frac{1}{(1+x)\sqrt{x(2-x)}} dx = \int_0^1 f(x) dx$$

$$f(x) = \frac{1}{(1+x)\sqrt{2x+x^2}}$$

---

# Question65

**If $I_n = \int_0^{\pi/4} \tan^n x dx$, then $\frac{1}{I_2+I_4} + \frac{1}{I_3+I_5} + \frac{1}{I_4+I_6} =$**

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**Options:**

A. $\frac{1}{I_9+I_{11}}$

B. $\frac{1}{I_{10}+I_{12}}$

C. $\frac{1}{I_{12}+I_{14}}$

D. $\frac{1}{I_{11}+I_{13}}$

**Answer: D**

## Solution:

$$I_n = \int_0^{\pi/4} \tan^n x dx$$

$$I_{n+2} = \int_0^{\frac{\pi}{4}} \tan^{n+2} x dx$$

Now, $I_n + I_{n+2} = \int_0^{\frac{\pi}{4}} \tan^n x dx + \int_0^{\frac{\pi}{4}} \tan^n x \tan^2 x dx$

$= \int_0^{\frac{\pi}{4}} \tan^n x \left(1 + \tan^2 x\right) dx = \int_0^{\frac{\pi}{4}} \tan^n x \sec^2 x dx$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\therefore I_n + I_{n+2} = \int_0^1 t^n dt = \frac{1}{n+1}$

Now, $\frac{1}{I_2 + I_4} + \frac{1}{I_3 + I_5} + \frac{1}{I_4 + I_6}$

$= \frac{1}{1/3} + \frac{1}{1/4} + \frac{1}{1/5}$

$= 3 + 4 + 5 = 12 = \frac{1}{I_{11} + I_{13}}$

---

# Question66

$\int_0^{\pi/4} e^{\tan^2 \theta} \sin^2 \theta \tan \theta d\theta =$

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**Options:**

A. $\frac{1}{2}\left(\frac{e}{2} - 1\right)$

B. $\frac{e}{2} - 1$

C. $\frac{\pi}{2}$

D. $2\left(\frac{\pi}{2} - e\right)$

**Answer: A**

## Solution:

Let $I = \int e^{\tan^2 \theta} \sin^2 \theta \tan \theta d\theta$

$= \int e^{\tan^2 \theta} \times \sin^2 \theta \frac{\tan \theta \sec^2 \theta}{\left(1 + \tan^2 \theta\right)} d\theta$

$= \frac{1}{2} \int 2 e^{\tan^2 \theta} \times \frac{\tan^2 \theta}{\left(1 + \tan^2 \theta\right)} \times \frac{\tan \theta \sec^2 \theta}{\left(1 + \tan^2 \theta\right)} d\theta$

Let $\tan^2 \theta = t$

$\Rightarrow 2 \tan \theta \sec^2 \theta d\theta = dt$

When $\theta = 0$, then $t = 0$

and when $\theta = \frac{\pi}{4}$, then $t = 1$

$$\therefore I = \frac{1}{2} \int_0^1 e^t \times \frac{t}{(1+t)^2} dt = \frac{1}{2} \int_0^1 e^t \left[\frac{1+t-1}{(1+t)^2}\right] dt$$
$$= \frac{1}{2} \int_0^1 e^t \left[\frac{1}{1+t} + \left(\frac{-1}{(1+t)^2}\right)\right] dt$$

$$= \frac{1}{2}\left[\frac{e^t}{1+t}\right]_0^1 \left[\because \int e^x \left(f(x) + f'(x) dx\right) = e^x f(x) + C\right]$$
$$= \frac{1}{2}\left[\frac{e}{2} - 1\right]$$

---

# Question67

$\int_{\pi/4}^{5\pi/4} (|\cos t| \sin t + |\sin t| \cos t) dt =$

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**Options:**

A. 0

B. 1

C. 1/2

D. $\sqrt{3}/2$

**Answer: A**

## Solution:

$\int_{\pi/4}^{5\pi/4} (|\cos t| \sin t + |\sin t| \cos t) dt$

$$= \int_{\pi/4}^{5\pi/4} |\cos t| \sin t + \int_{\pi/4}^{5\pi/4} |\sin t| \cos tdt$$

$$= \frac{2}{2} \int_{\pi/4}^{\pi/2} \cos \sin tdt + \frac{2}{2} \int_{\pi/2}^{5\pi/4} (-\cos t) \sin tdt$$

$$+\frac{2}{2} \int_{\pi/4}^{\pi} \sin t \cos tdt + \frac{2}{2} \int_{\pi}^{5\pi/4} (-\sin t) \cos tdt$$

$$= \frac{1}{2} \int_{\pi/4}^{\pi/2} \sin 2tdt - \frac{1}{2} \int_{\pi/2}^{5\pi/4} \sin 2tdt + \frac{1}{2} \int_{\pi/4}^{\pi} \sin 2tdt$$

$$- \frac{1}{2} \int_{\pi}^{5\pi/4} \sin 2tdt$$

$$= \frac{1}{2} \left[\frac{-\cos 2t}{2}\right]_{\pi/4}^{\pi/2} - \frac{1}{2} \left[\frac{-\cos 2t}{2}\right]_{\pi/2}^{5\pi/4} + \frac{1}{2} \left[\frac{-\cos 2t}{2}\right]_{\pi/4}^{\pi}$$

$$- \frac{1}{2} \left[\frac{-\cos 2t}{2}\right]_{\pi}^{5\pi/4}$$

$$= \frac{-1}{4} \left(\cos \pi - \frac{\cos \pi}{2}\right) + \frac{1}{4} \left(\cos \frac{5\pi}{2} - \cos \pi\right)$$

$$\frac{-1}{4} \left(\cos 2\pi - \frac{\cos \pi}{2}\right) + \frac{1}{4} \left(\cos \frac{5\pi}{2} - \cos 2\pi\right).$$

$$= \frac{-1}{4}(-1 - 0) + \frac{1}{4}(0 - (-1)) - \frac{1}{4}(1 - 0) + \frac{1}{4}(0 - 1)$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{1}{4} - \frac{1}{4} = 0$$

---

# Question68

**If $f(x) = \max\{\sin x, \cos x\}$ and $g(x) = \min\{\sin x, \cos x\}$, then $\int_0^\pi f(x)dx + \int_0^\pi g(x)dx =$**

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**Options:**

A. $2\sqrt{2} + 2$

B. $2\sqrt{2} - 2$

C. 2

D. $2\sqrt{2}$

**Answer: C**

## Solution:

Given, $f(x)=\max\{\sin x, \cos x\}$ and $g(x)=\min\{\sin x, \cos x\}$

$$\therefore = \int_0^\pi f(x)dx + \int_0^\pi g(x)dx = \int_0^\pi \sin dx + \int_0^\pi \cos x dx$$
$$= -[\cos x]_0^\pi + [\sin x]_0^\pi = -(-1-1)+0 = 2$$

---

# Question69

$\int_0^1 a^k x^k dx =$

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**Options:**

A. $\lim\limits_{n\to\infty} \frac{a^k(1+2^k+3^k\ldots+n^k)}{n^{k+1}}$

B. $\lim\limits_{n\to\infty} \frac{a^k+a^k+\ldots+a^k}{n^{k+1}}$

C. $\lim\limits_{n\to\infty} \frac{1}{n}\Sigma\left(\frac{r}{n}\right)^k$

D. $\lim\limits_{n\to\infty} \frac{1}{n}\Sigma\left(\frac{2r}{n}\right)^k$

**Answer: A**

## Solution:

If we consider the option (a), we have

$$\lim_{n\to\infty} \frac{a^k\left(1+2^k+3^k+\ldots.+n^k\right)}{n^k+1}$$
$$= \lim_{n\to\infty} \frac{a^k\left(1^k+2^k+3^k+\ldots..+n^k\right)}{n^k\cdot n}$$
$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{a^k r^k}{n^k}\cdot\frac{1}{n}$$
$$= \lim_{n\to\infty} \sum_{r=1}^{n} a^k\left(\frac{r}{n}\right)^k\cdot\frac{1}{n}$$

On putting $\frac{1}{n} \rightarrow dx, \frac{r}{n} \rightarrow x$ and $\Sigma \rightarrow \int$

$= \int_0^1 a^k \cdot x^k \cdot dx$

---

# Question70

**Let $\alpha$ and $\beta(\alpha < \beta)$ are roots of $18x^2 - 9\pi x + \pi^2 = 0, f(x) = x^2, g(x) = \cos x$. Then, $\int_\alpha^\beta x(g \circ f(x))dx =$**

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**Options:**

A. $\frac{\sqrt{3}-1}{4}$

B. $\frac{\sqrt{3}}{4}$

C. $\frac{2+\sqrt{3}}{2}$

D. $\frac{1}{2}\left(\sin \frac{\pi^2}{9} - \sin \frac{\pi^2}{36}\right)$

**Answer: D**

## Solution:

$f(x) = x^2, g(x) = \cos x$
$18x^2 - 9\pi x + \pi^2 = 0$

$$\Rightarrow \quad x = \frac{9\pi \pm \sqrt{81\pi^2 - 72\pi^2}}{36}$$

$$\Rightarrow \quad x = \frac{9\pi \pm 3\pi}{36}$$

$$x = \frac{12\pi}{36}$$

$$\Rightarrow \quad \beta = \frac{\pi}{3}$$

and $x = \frac{6\pi}{36} \Rightarrow \alpha = \frac{\pi}{6} \quad [\because \alpha < \beta]$

Now, $g \circ f(x) = g\left(f(x) = g\left(x^2\right) = \cos x^2\right.$

$$\therefore \int_{\alpha}^{\beta} x(g \circ f(x)) dx = \int_{\pi/6}^{\pi/3} x \cdot \cos x^2 dx$$

$$= \frac{1}{2} \int_{\pi/6}^{\pi/3} \cos\left(x^2\right) \cdot 2x dx$$

$$= \frac{1}{2}\left[\sin x^2\right]_{\pi/6}^{\pi/3}$$

$$= \frac{1}{2}\left[\sin \frac{\pi^2}{9} - \sin \frac{\pi^2}{36}\right]$$

---

# Question71

$\int_0^\pi x\left(\sin^2(\sin x) + \cos^2(\cos x)\right) dx =$

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**Options:**

A. $\pi^2$

B. $\pi^2/2$

C. $2\pi$

D. $\pi/4$

**Answer: B**

## Solution:

Let $I = \int_0^\pi x\left(\sin^2(\sin x) + \cos^2(\cos x)\right) dx$ ....(i)

$I = \int_0^\pi (\pi - x)\left(\sin^2(\sin x) + \cos^2(\cos x)\right) dx$ ... (ii)

Adding Eqs. (i) and (ii), we get

$$2I = \pi \int_0^{\pi} \left(\sin^2(\sin x) + \cos^2(\cos x)\right) dx$$

$$2I = 2\pi \int_0^{\pi/2} \left(\sin^2(\sin x) + \cos^2(\cos x)\right) dx$$

$$I = \pi \int_0^{\pi/2} \sin^2(\sin x) + \int_0^{\pi/2} \cos^2(\cos x) dx$$

$$I = \pi \int_0^{\pi/2} \sin^2(\cos x) + \int_0^{\pi/2} \cos^2(\cos x) dx$$

$$= \pi \int_0^{\pi/2} 1 dx$$

$$I = \pi \times \frac{\pi}{2}$$

$$I = \frac{\pi^2}{2}$$

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# Question72

**If $\int\limits_0^{\pi} \log(\sin x) dx = 8k$, then $\int\limits_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$ is equal to**

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**Options:**

A. $k$

B. $-k$

C. $\frac{k}{2}$

D. $4k$

**Answer: B**

## Solution:

$\int_0^{\pi} \log(\sin x) dx = 8k$

$$\text{Let } I=\int_0^{\pi/4}\log(1+\tan x)dx=\int_0^{\pi/4}\log(1+\tan\theta)d\theta$$
$$=\int_0^{\pi/4}\log(1+\tan(\pi/4-\theta))d\theta$$
$$I=\int_0^{\pi/4}\log\left(\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}\right)d\theta$$
$$I=\int_0^{\pi/4}\log 2d\theta-\int_0^{\pi/4}\log(1+\tan\theta)d\theta$$
$$I=\int_0^{\pi/4}\log 2d\theta-I\Rightarrow 2I=(\log 2)\left(\frac{\pi}{4}\right)$$
$$I=\frac{(\log 2)\pi}{8}=\frac{-1}{4}\times\left(\frac{-\pi}{2}\log 2\right)=-\frac{1}{8}(-\pi\log 2)$$
$$=\frac{-1}{8}\times 8k\quad\left[\begin{array}{l}\because\int_0^{\pi/2}\log\sin x dx=\frac{-\pi}{2}\log 2\\ =\int_0^{\pi}\log\sin x=-\pi\log 2\end{array}\right]$$
$$\therefore\quad I=-k$$

---

# Question73

**If $\int\limits_0^1 x^m(1-x)^n dx=k\int\limits_0^1 x^n(1-x)^m dx$, then the value of $k$ equals**

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**Options:**

A. $m$

B. $n$

C. $\frac{1}{\mathrm{mn}}$

D. 1

**Answer: D**

## Solution:

$$\int_0^1 x^n(1-x)^n dx = \int_0^1 (1-x)^n\,(1-(1-x)^n dx$$
$$= \int_0^1 (1-x)^n (x)^n dx \Rightarrow k = 1$$

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# Question74

**If $\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$, then the value of a is equal to**

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**Options:**

A. 1

B. 2

C. 3

D. 4

**Answer: B**

## Solution:

$$\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$$
$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\Big|_0^a = \frac{\pi}{8} \Rightarrow \tan^{-1}\left(\frac{a}{2}\right) = \frac{\pi}{4}$$
$$\Rightarrow \quad a/2 = 1 \Rightarrow a = 2$$

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# Question75

**$\int_1^2 \frac{x^3-1}{x^2}$ is equal to**

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**Options:**

A. $\frac{5}{3}$

B. $\frac{3}{5}$

C. 1

D. $-1$

**Answer: C**

## Solution:

$\int_1^2\left(\frac{x^3-1}{x^2}\right) dx=\left.\frac{x^2}{2}+\frac{1}{x}\right|_1^2=\frac{5}{2}-\frac{3}{2}=1$

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# Question76

**If $\int_0^{\pi / 2} \tan ^n(x) d x=k \int_0^{\pi / 2} \cot ^n(x) d x$, then**

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**Options:**

A. $k=1$

B. $k=2$

C. $k=\frac{1}{2}$

D. $k=3$

**Answer: A**

## Solution:

$$\int_0^{\pi/2} \tan^n x dx = k \int_0^{\pi/2} \cot^n(x) dx$$
$$\Rightarrow \int_0^{\pi/2} \tan^n \left(\frac{\pi}{2} - x\right) dx = k \int_0^{\pi/2} \cot^n(x) dx$$
$$\left\{\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx\right\}$$
$$\Rightarrow \int_0^{\pi/2} \cot^n x dx = k \int_0^{\pi/2} \cot^n x dx$$
$$k = 1$$

---

# Question77

**$\int_0^2 xe^x dx$ is equal to**

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**Options:**

A. $e^2 + 1$

B. $e^2 - 1$

C. $e^{-1} - 1$

D. $e^{-1} + 1$

**Answer: A**

## Solution:

$$\int_0^2 xe^x dx = (xe^x - e^x)_0^2$$
$$= (2e^2 - e^2) - (0.e^0 - e^0) = e^2 + 1$$

---

# Question78

**$\int_2^4 \{|x-2| + |x-3|\} dx$ is equal to**

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**Options:**

A. 1

B. 2

C. 3

D. 4

**Answer: C**

## Solution:

$$\text{Let } I=\int_2^4(|x-2|+|x-3|)dx$$

$$\because x-a=\begin{cases} x-a, & x\geq a \\ -(x-a), & x<a \end{cases}$$

$$\therefore \quad I=\int_2^4|x-2|dx+\int_2^3|x-3|dx+\int_3^4|x-3|dx$$

$$=\int_2^4(x-2)dx-\int_2^3(x-3)dx+\int_3^4(x-3)dx$$

$$=\left(\frac{x^2}{2}-2x\right)_2^4-\left(\frac{x^2}{2}-3x\right)_2^3+\left(\frac{x^2}{2}-3x\right)_3^4$$

$$=\left[\left(\frac{16}{2}-8\right)-\left(\frac{4}{2}-4\right)\right]-\left[\left(\frac{9}{2}-9\right)-\left(\frac{4}{2}-6\right)\right]$$

$$=\left|[0-(-2)]-\left[\left(\frac{-9}{2}\right)-(-4)\right]+\left[(-4)-\left(-\frac{9}{2}\right)\right]\right|$$

$$=\left|2+\frac{9}{2}-4-4+\frac{9}{2}\right|=[2+9-8]=3$$

---

# Question79

$$\int\limits_{-1/2}^{1/2}\left\{[x]+\log\left(\frac{1+x}{1-x}\right)\right\}dx \textbf{ is equal to}$$

**Options:**

A. $2\log(1/2)$

B. 0

C. $\frac{-1}{2}$

D. 1

**Answer: C**

## Solution:

(c) Let $I = \int_{-1/2}^{1/2}\left\{[x] + \log\left(\frac{1+x}{1-x}\right)dx\right\}$

$\because \int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & f(-x) = f(x) \\ 0, & f(-x) = -f(x) \end{cases}$

$= \int_{-1/2}^{1/2}[x]dx + \int_{-1/2}^{1/2}\log\left(\frac{1+x}{1-x}\right)dx$

Let $f(x) = \log\left(\frac{1+x}{1-x}\right)$

$f(-x) = \log\left(\frac{1-x}{1+x}\right) = -f(x) \qquad$ [odd function]

$= \int_{-1/2}^{1/2}[x]dx + 0 = \int_{-1/2}^{0}[x]dx + \int_{0}^{1/2}[x]dx$

$= \int_{-1/2}^{0}(-1)dx + \int_{0}^{1/2}0dx$

$= -(x)_{-1/2}^{0} + 0 = -\left(0 + \frac{1}{2}\right) = -\frac{1}{2}$

---